Available online at http://www.journalijdr.com

International Journal of
DEVELOPMENT RESEARCH

ISSN: 2230-9926

International Journal of Development Research
Vol. 06, Issue, 11, pp.10331-10332, November, 2016

Full Length Research Article

LEFT REGULAR BI NEAR SUBTRACTION SEMIGROUPS

[1]Firthous Fatima, S. and [2]Jayalakshmi, S.

[1]Department of Mathematics, Sadakathullah Appa College (Autonomous), Thirunelveli- 627 011, Tamil Nadu, India,
[2]Department of Mathematics, Sri Parasakthi College (Autonomous) for women, Courtallam, Tamil Nadu, India

ARTICLE INFO

Article History:

Received 17th August, 2016
Received in revised form
03rd September, 2016
Accepted 14th October, 2016
Published online 30th November, 2016

Key Words:

Left Regular- bi near subtraction semigroups, Right Regular- bi near subtraction semigroups, Regular-near subtraction semigroups, Left Strongly Regular-near subtraction semigroups, Right Strongly Regular-near subtraction semigrousp, Nil near subtraction semigroup.

ABSTRACT

In this paper we introduce the notion of Left Regular- bi-near subtraction semigroup. Also we give characterizations of Left Regular- bi-near subtraction semigroup.

INTRODUCTION

In [3,4], Y. V. Reddy and C. V. L. N. Murty, has introduced, On Strongly Regular Near Rings A near subtraction semigroup X is regular if for all x∈X, there exists a∈X with x= xax. A near subtraction semigroup X is left strongly regular if for all x∈X, there exists a∈X with $x=ax^2$. A near subtraction semigroup X is right strongly regular if for all x∈X, there exists a∈X with $x=x^2a$. Motivated by these concepts we introduce left regular bi near subtraction We obtain some characterisation of Left Regular bi near subtraction semigroup

Preliminaries

A non-empty subset X together with two binary operations "–" and "." is said to be subtraction semigroup If (i) (X,–) is a subtraction algebra (ii) (X, .) is a semi group (iii) x(y–z)=xy–xz and (x–y)z= xz–yz for every x, y, z∈X. A non-empty subset X together with two binary operations "–" and "." is said to be near subtraction semigroup if (i) (X,–) is a subtraction algebra (ii) (X,.) is a semi group and (iii) (x–y)z= xz–yz for every x, y, z∈X. A non-empty subset X is said to be S_1-near subtraction semigroup if for every a∈X there exists $x∈X^*$ such that axa=xa. A non-empty subset X is said to be **S_2-** near subtraction semigroup if for every a∈X there exists $x∈X^*$ such that axa=ax. A non-empty subset X is said to be nil-near subtraction semigroup if there exists a positive integer k 1 such that $a^k=0$ Which implies that xa=0 where $x=a^{k-1}$.

In this section, We establish new concept of left regular bi near subtraction semigroup and some properties of left regular bi near subtraction semigroup

Definition 3.1.1

A near subtraction semigroup X is regular if for all x∈X, there exists a∈X with x= xax

****Corresponding author: Firthous Fatima,***
Department of Mathematics, Sadakathullah Appa College (Autonomous), Thirunelveli- 627 011, Tamil Nadu, India

Definition 3.1.2

A near subtraction semigroup X is left strongly regular if for all x∈X, there exists a∈X with $x=ax^2$

Definition 3.1.3

A near subtraction semigroup X is left regular bi near subtraction semigroup. if X is both regular and left strongly regular near subtraction semigroup.

Example 3.1.4

Let X={0,a,b,1} in which "-" and "." be defined by

-	0	a	b	1
0	0	0	0	0
a	a	0	a	0
b	b	b	0	0
1	1	b	a	0

.	0	a	b	1
0	0	0	0	0
a	0	a	b	1
b	0	b	b	a
1	0	1	a	1

Then X is a left regular bi near subtraction semigroup

Definition 3.1.5

A near subtraction semigroup X is right strongly regular if for all x∈X, there exists a∈X with $x=x^2a$.

Definition 3.1.6

A near subtraction semigroup X is right regular bi near subtraction semigroups. if X is both regular and right strongly regular near subtraction semigroup.

Example 3.1.7

Let X={0,a,b,c} in which "-" and "." be defined by

-	0	a	b	c
0	0	0	0	0
a	a	0	a	0
b	b	b	0	0
c	c	b	a	0

.	0	1	2	3
0	0	0	0	0
a	a	a	a	a
b	b	b	b	b
c	c	c	c	c

Then X is a right regular bi near subtraction semigroup

Definition 3.1.8

A near subtraction semigroup X is said to be reduced if it has no nilpotent elements

Definition 3.1.9

A near subtraction semigroup X is said to be IFP if ab=0⇒axb=0 for allx∈X.

Lemma 3.1.10

(a) if X is left(right)strongly regular, it is reduced.
(b) In zero-symmetric reduced near subtraction semigroup, ab=0 ⇒ ba=0, and IFP holds.

Proof: (a) The right strongly regular case is trivial. If $x^2=0$ and $x=ax^2=a0$ then $0=x^2$ $=(a.0)x= a(0x)=a0=x$. (b) The proof is obvious.

Lemma 3.1.11

A left regular bi near subtraction semigroup with IFP is right regular bi near subtraction semigroup

Proof:

If $x=ax^2=xax$, then $(ax-xa)x=0$so by IFP $(ax-xa)ax=0$ and similarly we have $(xa-ax)ax=0$. Therefore $axax=xa^2x$. Thus $ax=axax=xa^2x$ so $x=xax=x^2a^2x$. (ie.,) $x=x^2b$ where $b= a^2x$. Moreover $xbx=x\ a^2x.x=xax =x$.

Example 3.1.12

Converse of the above Lemma need not be true

Let X={0,1,2,3} in which "-" and "." be defined by

-	0	1	2	3
0	0	0	0	0
1	1	0	1	0
2	2	2	0	0
3	3	2	1	0

.	0	1	2	3
0	0	0	0	0
1	1	1	1	1
2	2	2	2	2
3	3	3	3	3

This X is a right regular bi near subtraction semigroup but not a left regular bi near subtraction semigroup [Since $a\neq ba^2$, $a\neq ca^2$]

Proposition 3.1.13

If x is zero symmetric, left regular is equivalent to left strong regular and these imply right regular. Moreover if X is unital, all three conditions are equivalent.

Proof:

If X is left strongly regular, then for all x∈X there exists a ∈X such that $x=ax^2$. It demands that $(x-xax)x=x^2-xax^2= x^2- x^2=0$. So by lemma 3.1.10, Also, $x(xax-x)=0$ Then $(x-xax)^2=x(x-xax)-xax(x-xax)=0.$⇒ $x-xax=0${Since X is reduced]. Similarly we can prove $xax-x=0$. Therefore $x=xax$. Thus X is left regular. Then X is right regular [by the lemmas3.1.10 and 3.1.11]. if X is unital and $x=x^2a=xax$, then xa and ax are idempotents. Now, $x = xax =x(ax)=(ax)a =ax^2$. Thus X is a Strongly regular.⇒ X is left regular.

REFERENCES

[1] Dheena, P. and G. Sathesh Kumar. 2007. *On Strongly Regular Near- Subtracttion semi groups*, *Commun. Korean Math. Soc.* 22, No.3, pp.323-330.
[2] Pilz Gunter, 1983. *Near-Rings*, North Holland, Amsterdam.
[3] Reddy, Y.V. and Murty, C.V.L.N. 1984. On Strongly Regular Near Rings, Procedings of the Edinburgh Mathemtical Society, 27,61-64.
[4] Reddy, Y.V. and Murty, C.V.L.N. 1985. A note on stronlgy regular near-rings. Publication of Mathematical Japonicae Decbrecary.
[5] Von Neumann. J. 1936. On regular ring, *Proc. Nat. Acad. USA*. 22, 707-713.
[6] Zelinka, B.1995. Subtraction semigroups, *Math. Bohemica,* 120, 445-447.

****