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MODELING VOLATILITY'S LONG-RANGE PERSISTENCE AND ASYMMETRY EFFECT OF BRADESCO BANK STOCK PRICES USING GARCH MODELS

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ABSTRACT

The main aim of this paper is to evaluate and model both leverage effect and persistent volatility of Bradesco Bank stock shares (BBDC3). The leverage effect was measured through the generalized autoregressive conditional heteroscedasticity (GARCH) model and some of its extensions, such as EGARCH, NGARCH, APGARCH, ALLGARCH, IGARCH, CGARCH and FIGARCH. The orical basis: These are mainly asymmetrical extensions that were chosen since they can overcome the possible limitation that negative returns might have a bigger impact in volatility than positive ones in this type of data. Results indicated that the most common used goodness-of-fit information criteria might not be a sufficient measurement for comparing different kinds of GARCH models for forecasting and, for this reason, it might be necessary to consider other important volatility characteristics, such as long-range persistence. In BBDC3 returns, despite CGARCH model slightly overestimated the central tendency, it still significantly outperformed the asymmetric extensions of GARCH model.

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INTRODUCTION

Financial time series have been gaining more notoriety in the time series research area, mainly due to the necessity of knowing the main risks of the financial market, which is an environment of great uncertainty. For Robinson *et al.* (2018), stock exchanges are, by definition, barometers of investor and public sentiment. For these authors, the collective value-perception of the investing public is very important for the value of securities in the international stock market. Since stock prices are usually close to random walk processes, price returns have small or no autocorrelation and, therefore, are not suitable for being modeled by autoregressive moving average (ARMA) models (BOX AND JENKINS, 1970). Nevertheless, the variance of these returns commonly presents patterns of temporal dependence that smoothly oscillates, and it is easier to be modeled. For this reason, researchers began to study and model the so-called volatility – defined as the conditional standard deviation, Morettin (2011), instead of prices themselves. The study of financial market's volatility is important, as it is a central parameter for many financial decision-making such as the pricing and hedging of derivative products as well as the development of efficient risk management methods (ANÉ, 2006). In addition, according to Poon and Granger (2003), the volatility of the stock market can affect the economy as a whole. Global incidents like corruption scandals and economic crises may influence the stock markets of many countries and, therefore, affect the global economy. Hence, it is common for politicians and company managers to monitor the financial market behaviour, since volatility is a thermometer of economic fragility. Classical time series models (e.g., ARMA) are not suitable for modeling the volatility of price returns, since constant variance is one of their main assumptions and they are limited to the estimation of the first moment. One natural alternative is to use heteroscedastic models, which can handle the volatility as a time-varying parameter. In this sense, Engle (1982) pioneered the autoregressive conditional heteroscedasticity (ARCH) model, which was created for modeling the United Kingdom's inflation rate. Then, Bollerslev (1986), noticing that ARCH model commonly led to over parameterization, introduced a more parsimonious model known as generalized autoregressive conditional heteroscedasticity (GARCH) model, which has similar properties to Engle's model. GARCH models are widely used for modeling volatility, mainly in financial stock markets. According to Beg and Anwar (2014), these models are able to identify most of the volatility properties, as time-varying clusters and mean reversion. They can also accommodate fat-tailed (i.e., leptokurtic) distributions such as Student's t and generalized error (GED) distributions (NELSON, 1991), that more realistically describe the density of financial returns. Although GARCH models are useful to identify the above-mentioned properties, they also have some limitations, such as the inability to differentiate the impact of negative and positive shocks since it takes into consideration the squared returns to compute volatility. One of the first to document the possibility that negative returns can have a bigger impact in volatility than positive ones – namely leverage effect – was Black (1976), followed by many others as Christie (1982), Nelson (1991), Glosten*et al.* (1993), and Engle and Ng (1993). Further, in order to overcome this limitation, many asymmetric extensions of the GARCH model were proposed. Some of the most notorious extensions are the exponential GARCH (EGARCH) (NELSON, 1991), asymmetric Power GARCH (APGARCH) (DING*et al.*, 1993) and threshold GARCH (ZAKOIAN, 1994). Those models can be particularly useful in capturing the asymmetric effect and can lead to better forecast results. Besides the asymmetric effect, volatility also exhibits persistence (ENGLE AND MUSTAFA, 1992), which means that the effects of shocks take a long time to dissipate. Although this property is present in many financial time series, its effect usually last longer in foreign exchange rates (BEG AND ANWAR, 2014). It has been documented that standard GARCH models can overestimate the volatility when high persistence is present (ENGLE AND MUSTAFA, 1992; LAMOUREZ AND LASTRAPES, 1993), which motivated the development of models adapted to identify the volatility long-range persistence. For instance, Engle and Bollerslev (1986) proposed the integrated GARCH (IGARCH) model, which handles the volatility as an unit root process. Baillie*et al.* (1996) associated the long-range persistence with a long memory process, introducing the fractionally integrated GARCH (FIGARCH). More recently, Engle and Lee (1999) designed the component GARCH (CGARCH) that introduces a transitory component for considering the persistent volatility. Therefore, it is possible to notice that volatility has some complex properties that are not properly identified by standard GARCH models, such as the leverage effect and long-run dependence. In this context, GARCH extensions are important, since they can be used to handle better these properties and lead to better results. Volatility is an important parameter in financial applications, since it is a proxy for the risk, so it is crucial that all the properties of volatility be considered, to perform a more consistent modeling with reality.

Given all the previous information, the goal of this paper is to forecast the volatility of financial returns, focusing on the properties of leverage effect and high persistence. We also want to evaluate the performance of the GARCH model and its extensions: EGARCH, NGARCH, APGARCH, ALLGARCH, IGARCH, CGARCH and FIGARCH, in the modeling of time series represented by a distribution with very heavy tails, characterized as leptokurtic, in addition to symmetrical. The analysis was developed in the series of price returns of Bradesco Bank (BBDC3) for presenting the desired properties. In addition, this institution is one of the biggest banking services companies in Brazil and its shares have high liquidity (high number of trades), being representative of the Brazilian stock exchange. According to DIEESE (2020), the five largest banks in Brazil, including Bradesco, achieved 30.3% increase in their profits in 2019, when compared to the previous year. Bradesco grew 5.6% in its assets, reaching R$ R1.4 trillion at the end of 2019. In the same year, this bank had the greatest evolution in credit operations among the private banks, with an increase of 13,8%, reaching R$ 605.0 billion. Trindade *et al.* (2020) also show that the privileged position of this bank, which, together with the Brazilian Petroleum Corporation - Petrobras, Vale S.A. e Banco do Brazil, had stocks with higher trading in Stock Exchange and Over-the-Counter Market located at São Paulo – Brazil.

THEORETICAL BASIS

Volatility stylized facts: Many stylized facts about the volatility were observed over the years. These properties have been confirmed by numerous studies (ENGLE AND PATTON, 2001) and it is important that the heteroscedastic models are able to deal with these properties. One common volatility characteristic is that usually it exhibits high persistence, which means that past shocks affect future shocks many steps ahead. Volatility clusters explain this long-range dependence. Mandelbrot (1963) and Fama (1965) were some of the pioneers to present evidences that positive or negative shocks are followed by shocks of same sign and similar magnitude. Many others have also appointed this behavior, such as Baillie *et al.* (1996), Chou (1988) and Schwert (1989). The fact that volatility behaves in clusters means that it presents high intensity in some periods and low magnitude in others. Therefore, high volatility periods will eventually give place to more stable volatility and, then, to a low volatility period, followed by a volatility raise. Therefore, the volatility is mean reverting, which means that it will eventually go back to its average value, never diverging to infinite (ENGLE AND PATTON, 2001).

Another important stylized fact, firstly documented by Black (1976) and then confirmed by other researchers as Nelson (1991), and Engle and Ng (1993), is that positive and negative returns do not affect the volatility with the same impact. This asymmetry is called leverage or risk premium effect. According to Engle and Patton (2001), although leverage effect is commonly observed in financial stocks, it has not yet been reported that this asymmetric behavior is present in exchange rates. Lastly, it is important to consider that time series with high frequency, which is the case of financial returns, usually present heavy-tailed distributions. Therefore, these series have positive excess of kurtosis (i.e., leptokurtic shapes), implying that the distribution puts more mass on the tails of its support than a normal distribution does (TSAY, 2005). For this reason, it is common to use distributions as Student's t or generalized error distribution (GED) (NELSON, 1991) in the specification of GARCH models.

GARCH Model

Let r_t denote a series of daily price returns, given by

rt = pt-1 (1)

where p_t is the closing log-price corresponding to day t. Most of financial studies involve returns instead of prices, since they are scale-free and have more attractive statistical properties, such as stationarity and ergodicity (TSAY, 2005). The conditional mean and variance of the return process are given by

$$u_t = E(r_t | F_{t-1}),$$

$$h_t = V \quad (r_t \vee F_{t-1}) \qquad (2)$$

respectively, where F_{t-1} is all available information up to $t-1$. The general idea in modeling volatility is that the returns have a small or no autocorrelation, thus it is possible to consider that $u_t = 0$ and the conditional variance equation (Equation 2) can be approximate as

$$h_t = E(r_t^2 | F_{t-1}). \quad (3)$$

Therefore, the squared returns are a non-biased estimator of the volatility. Bollerslev (1986), considering this fact and continuing the work of Engle (1982), proposed the GARCH model. Mathematically, a GARCH (m, n) model is described by

$$r_t = \sqrt{h_t}\varepsilon_t,$$

$$h_t = \alpha_0 + \sum_{i-1}^{m} \alpha_i r_{t-i}^2 + \sum_{j-1}^{n} \beta_j h_{t-j}^2, \quad (4)$$

where ε_t is a white noise process, i.e., presents no correlation between its values at different times and is usually specified with a heavy-tailed distribution (e.g., Student's t), m is the lag of the past squared returns and n is the lag value corresponding to the past volatility. The conditions $\alpha_0 > 0$, $\alpha_0 \geq 0$, $\alpha_m \geq 0$, $\beta_0 \geq 0$ and $\beta_j \geq 0$ ensure the positive sign for the variance, whereas the condition for stationarity, both weak and strict, is $\sum_{i-1}^{q}(\alpha_i + \beta_i) < 1$, where $q = m \quad (m, n)$.

In financial applications, the conventional GARCH model has arguably been the most popular model for conditional variance and has proved to yield accurate forecasts in many applications (LANNE AND SAIKKONEN, 2005). However, as shown in Equation 3, the squared returns can only be used to compute the volatility if the conditional mean is equal to zero, which means that there should not be autocorrelation in the series under analysis. Nevertheless, in some cases, the returns may show signs of autocorrelation, which can violate some theoretical assumptions presented in Equation 3. In order to avoid this issue, it is common to build a two-step model: firstly we apply an autoregressive moving average (ARMA) (BOX AND JENKINS, 1970) model to remove the autocorrelation and secondly we apply the GARCH model to the obtained residuals (MORETTIN, 2011). An ARMA (p,q) model for a series of returns r_t is written as

$$r_t = c + \varepsilon_t + \sum_{i-1}^{p} \phi_i r_{t-i} + \sum_{j-1}^{q} \theta_j \varepsilon_{t-j} \quad (5)$$

where c is a constant and $\varepsilon_t N(0, \sigma^2).\varepsilon_t$ In order to simplify the nomenclature, from now on r_t can refer to a series of price returns or to the residuals of an ARMA model.

Asymmetric GARCH models: As mentioned before, many financial returns time series show signs of leverage effect, in which negative returns have more impact in the volatility than positive returns. Since the conventional GARCH model is not able to consider this property, some extensions were proposed to overcome this weakness. Those extended models handle the leverage effect in different manners. This paper focuses on the EGARCH, APGARCH and TGARCH models.

EGARCH model: Nelson (1991) introduced the EGARCH(m,n) model, which is described by

$$r_t = \sqrt{h_t}\varepsilon_t,$$
$$\log(t) = \alpha_0 + \frac{1+\beta_1 B + \cdots + \beta_{n-1}B^{n-1}}{1-\alpha_1 B - \cdots - \alpha_m B^m} g(\varepsilon_{t-1}), \quad (6)$$

where B is the back-shift operator, i.e., $B \quad (\varepsilon_t) = g(\varepsilon_{t-1})$, and $g(\varepsilon_t)$ is the weighted innovation given by

$$g(\varepsilon_t) = \begin{cases} (\theta+\gamma)\varepsilon_t - \gamma \quad (|\varepsilon_t|) \text{if } \varepsilon_t \geq 0, \\ (\theta-\gamma)\varepsilon_t - \gamma \quad (|\varepsilon_t|) \text{if } \varepsilon_t < 0 \end{cases} \quad (7)$$

It is possible to notice that in Equation 5 the logarithm of the conditional variance is used, which is an advantage of the EGARCH model, since we do not need to worry about the sign of the coefficients.

APGARCH model: The asymmetric power GARCH model (APGARCH) (DING*et al.*, 1993) is a modified version of the nonlinear GARCH (NGARCH) (HIGGINS AND BERA, 1992) model, with the difference that the latter does not consider the asymmetric effect. However, both models parametrize the volatility raised to a power as a function of the past volatility and past returns raised to the same power. An APGARCH (m, n) model can be written as

$$r_t = \sqrt{h_t}\varepsilon_t,$$

$$\sigma_t^{\delta} = \alpha_0 + \sum_{i-1}^{m} \alpha_i (|r_{t-i}|)^{\delta} + \sum_{j-1}^{n} \beta_j \sigma_{t-j}^{\delta}. \quad (8)$$

where δ is positive and the leverage effect is given by γ_i. Setting $\delta = 2$ and $\gamma_i = 0$we have the conventional GARCH model.

TGARCH model: The threshold GARCH (TGARCH) model, introduced by Zakoian (1994), unlike the APGARCH model, considers the symmetric effect while keeping the conditional variance as a linear function. This model is an extension of the Taylor-Schwert GARCH (TS-GARCH) (SCHWERT, 1989; TAYLOR, 1986) and is closely related to the Glosten-Jagannathan-Runkle GARCH (GJR-GARCH) (GLOSTEN *et al*., 1993). The TGARCH(m,n) model can be written as

$$r_t = \sqrt{h_t}\varepsilon_t$$

$$\sigma_t = \alpha_0 + \sum_{i-1}^{m} \alpha_i r_{t-i} + \sum_{j-1}^{n} \beta_j \sigma_{t-j} + \sum_{i-1}^{m} \gamma_i d_{t-i} r_{t-i} \tag{9}$$

where d_t is a dummy variable subject to conditions, i.e. $d_t = 1, \text{ìı}\, r_t < 0 \;\square\; d_t = 0, \text{ìı}\, r_t \geq 0$

GARCH models for highly persistent volatility: In many financial applications the estimates obtained for both parameters α and β of a GARCH model are such that their sum is relatively close to unity, nearly violating the stationarity condition $\sum_{i-1}^{q}(\boldsymbol{\alpha_i} + \boldsymbol{\beta_i}) < 1$(LANNE AND SAIKKONEN, 2005). In this case, the models may exaggerate the volatility persistence and lead to poor forecasts. In order to overcome this issue, some extensions can be used such as the IGARCH, FIGARCH and CGARCH models.

IGARCH model: The integrated GARCH (IGARCH) model proposed by Engle and Bollerslev (1986) imposes an exact unit root to the volatility process, considering $\sum_{i-1}^{q}(\alpha_i + \beta_i) = 1$. An IGARCH (1,1) model can be written as

$$r_t = \sqrt{h_t}\varepsilon_t$$

$$h_t = \alpha_0 + \beta_1 h_{t-1} + (1 - \beta_1) r_{t-1}^2, \tag{10}$$

where $0 < \beta_1 < 1$. Note that the IGARCH (1,1) definition is close to an exponential smoothing weighted by factor β_1.

FIGARCH model: Baille*et al.* (1996) introduced the fractionally integrated GARCH (FIGARCH) model, which associates the slow decaying of squared returns autocorrelations with a long memory process. According to Tsay (2005), a time series is a long memory process if its autocorrelation function decays at a hyperbolic, instead of an exponential, rate as the lag increases.

A GARCH (m,n) model can be written as an ARMA(q,n) process, setting $q = m \quad (m, n)$, i.e.

$$\square(B) r_t^2 = \alpha_{U} + \beta(B) v_t, \tag{11}$$

where $\square(B) = 1 - \square_1 B - \cdots - \square_q B^q$ and $\beta(B) = 1 - \beta_1 B - \cdots - \beta_n B^n$ are polynomial functions and $v_t = r_t^2 - h_t$ is a martingale process, i.e., with dependent variances. By introducing a fractional operator d into Eq.8, we obtain the equation for a FIGARCH (q, n) model, given by

$$\square(B)(1 - \beta)^d r_t^2 = \alpha_{U} + \beta(B) v_t, \tag{12}$$

The term fractionally comes from the fact that a FIGARCH model considers an integration factor belonging to the range $0 < d < 1$.

CGARCH model: The component GARCH (CGARCH) model, pioneered by Lee and Engle (1999), decomposes the conditional variance into a permanent and transitory component in order to investigate the long and short run movements of volatility. A CGARCH (m, n) can be described as

$$r_t = \sqrt{h_t}\varepsilon_t$$

$$h_t = \xi_t + \sum_{i-1}^{m} \alpha_i (r_{t-i}^2 - \xi_{t-i}) + \sum_{j-1}^{n} \beta_j (h_{t-j} - \xi_{t-j})$$

$$\xi_t = \alpha_0 + \rho \xi_{t-1} + \phi(r_{t-1}^2 - h_{t-1}) \tag{13}$$

where ξ_t is the permanent component and the difference between the conditional variance and its trend and $h_{t-j} - \xi_{t-j}$is the transitory component of the conditional variance.

Forecast assessment: Usually in the application of time series models such as ARMA, residuals are calculated by the difference between observed and estimated values, and then the forecast errors are computed. However, volatility is a latent measurement, which makes this approach impossible to be conducted directly, since it is necessary to first choose a proxy for volatility (ANDERSEN AND BOLLERSLEY, 1998). An intuitive choice would be obtaining the absolute or squared returns, seeing that GARCH models estimate the conditional variance from returns data. Although the squared returns provide an unbiased estimate for volatility, this proxy can produce highly noisy measurements, due the error term ε_t (PEROBELLI*et al.*, 2013). In fact, according to Andersen and Bollerslev (1998), this component presents a high degree of variation in comparison to component h_t^2, which makes a large portion of this variable to be attributed to random shocks (ε_t). Hence, the GARCH model may result in poor forecast results when judged by absolute or squared returns.

Therefore, it is crucial to select a suitable proxy for volatility. Andersen and Bollerslev (1998) proposed to calculate the so-called realized volatility from intraday returns, given by $r_{d,m} = p_{d,m} - p_{d,m-1}, m = 2, \ldots, M, d = 1, \ldots, D,$where d and m refer to the day and number of observations, respectively, which can provide a more robust proxy for volatility. Further, realized variance for day d is defined by

$$R\ _d = r_{d,1}^2 + \sum r_{d,m}^2, d = 1, \ldots, D, \tag{14}$$

where $r_{d,1} = p_t - p_{t-1}$ is the overnight returns. Finally, the realized volatility for a day d is given by

$$R \quad _d = \sqrt{R \quad d} \tag{15}$$

In order to provide a better understanding of the realized volatility concept, Figure 1 provides a timeline schema of the Brazilian Stock Exchange (BOVESPA).

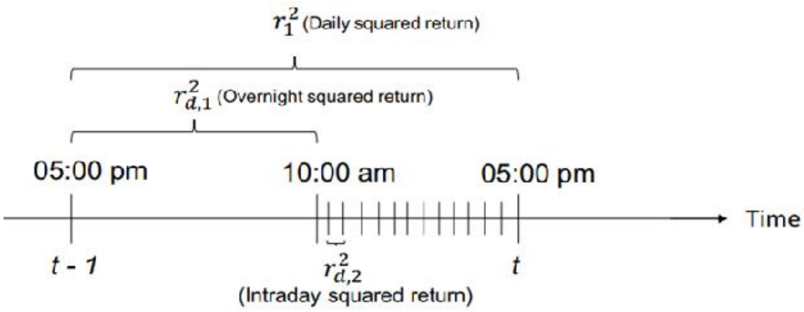


Figure 1. Timeline schema of the Brazilian Stock Exchange (BOVESPA). Source: Adapted from Hansen and Lunde (2005)

With the realized volatility determined from intraday data, it is possible to calculate the forecast errors through loss functions. Three widely used are the mean absolute error (MAE), root mean square error (RMSE) and mean absolute percentage error (MAPE), which are defined

$$M \quad = L^{-1} \sum_{t-1}^{L} |R \quad _t - h_t|, \tag{16}$$

$$R \quad = L^{-1} \sqrt{\sum_{t=1}^{L} (R \quad _t - h_t)^2}, \tag{17}$$

$$M \quad = L^{-1} \sum_{t=1}^{L} \frac{(R \quad _t - h_t)}{h_t} \times 100. \tag{18}$$

respectively, where L represents the L-steps ahead forecast horizon.

METHODOLOGY

Bradesco bank's stock prices (BBDC3) were obtained from InfoMoney database (INFOMONEY,2018) and consist of daily prices from February 1st, 2007 to November 4th, 2018, adding up to 2,795 observations to fit the volatility model. The range from December 4th, 2018 to December 5th, 2018 is the forecast horizon, which consists of 21 observations for the validation sample. For this last period, the realized volatility was calculated using 15 minutes intraday returns for each day, in order to compare with the forecast results obtained by the GARCH models.

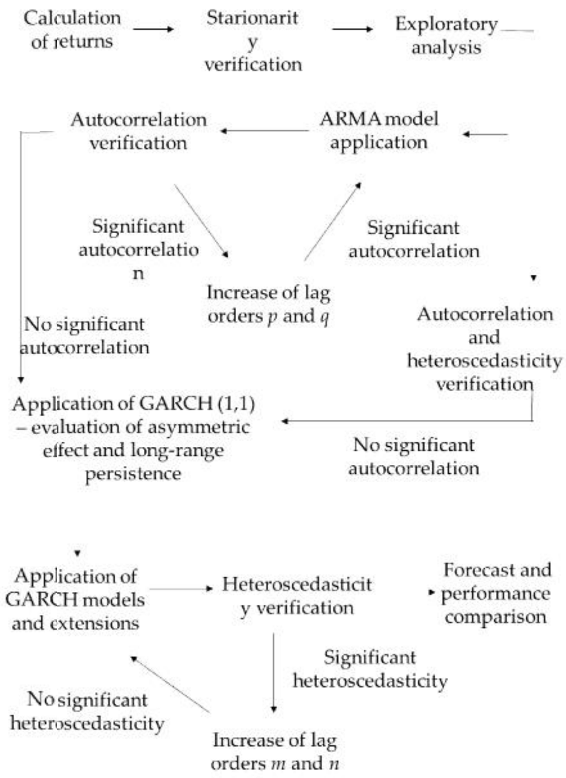
The methodology applied to this study is as follows:

1. Exploratory data analysis. Firstly, returns are calculated from the stock prices and an exploratory analysis is performed. Then, the stationarity and autocorrelation of the return process are evaluated through, respectively, the augmented Dickey-Fuller (ADF) (DIKEY AND FULLER, 1981) and Ljung-Box (LB) (LJUNG AND BOX, 1978) tests. If the returns time series is found to have significant autocorrelation, then an ARMA model must be performed before the application of a GARCH model.
2. ARMA model specification. If the autocorrelation is significant, then an ARMA (p,q) model is applied. In order to define both p and q orders, all possible combinations from orders (1,0) and (0,1) up to (6,6) are tested. The desirable model is chosen by the Akaike information (AIC) (AKAIKE, 1973), Bayesian information (BIC) (SCHWARZ, 1978) and Hannan-Quinn information (HQIC) (HANNAN AND QUINN, 1979) criteria. Additionally, the selected model must generate residuals without significant autocorrelation, which is checked again by the LB test. If autocorrelation is found to be significant, then the ARMA lag orders must be increased until autocorrelation is eliminated. Finally, the ARCH-LM (ALM) test (ENGLE, 1982) is performed in order to verify whether the selected ARMA model residuals have significant heteroscedasticity and hence justifying the application of a GARCH model.
3. Application of GARCH model. Similarly, to the ARMA models procedure of selection, the best GARCH model is chosen through the AIC, BIC and HQIC. However, in this case, the orders m and n are defined testing all possible combinations up to order (2,2), since usually GARCH models lead to low lag orders (MORETTIN, 2011). The presence of highly persistent volatility is evaluated by the GARCH (1,1) model coefficients α and β and the asymmetry effect is checked through the size and bias tests (ENGLE AND NG, 1993) in GARCH residuals. Since volatility is a latent measurement, the residuals cannot be computed directly. Hence, it is common to use the standard residuals $\tilde{r}_t = r_t / \sqrt{h_t}$. The process $\tilde{r}_t$ must not present significant heteroscedasticity, which can be verified by the ARCH-LM test. If heteroscedasticity is still present, then the GARCH orders must be increased until this effect is eliminated. All GARCH models in this study are specified following the Student's t distribution, hence the residuals should follow this specification as well. This is verified through simulated envelopes (ATKINSON, 1985).
4. Application of EGARCH, APGARCH and TGARCH. Independently on size and bias test's results, the asymmetric models EGARCH, APGARCH and TGARCH are performed. The idea is to check whether the tests provide useful information about the leverage effect in the time series under analysis. As for the lag orders, these are defined with identical procedure to that used in the previous item. In

addition, the ARCH-LM test is executed in order to check whether the models are able to remove the significant heteroscedasticity. Finally, the news impact curves (NIC) (ENGLE AND NG, 1993) are built in order to verify how these models handle the negative and positive returns.

5. Application of IGARCH, FIGARCH and CGARCH. Similarly to the previous phase, models for persistent volatility – IGARCH, FIGARCH and CGARCH – are executed, independently on the result of the sum $\sum_{i-1}^{q}(\alpha_i + \beta_i)$. The best models are once again selected with the previously mentioned information criteria through the same procedure used in items 3 and 4. Furthermore, the ARCH-LM test is performed to check whether these models are capable of removing the significant heteroscedasticity.
6. Forecast and performance comparison. The best GARCH, EGARCH, APGARCH, TGARCH, IGARCH, FIGARCH and CGARCH models, chosen through the information criteria, are applied in forecasting. The forecast evaluation and performance comparison are made through the loss functions MAE, RMSE and MAPE.

All analyses are performed within the R software (R CORE TEAM, 2019), using rugarch, tseries, Atsa, FinTS and urca packages, among others. Figure 2 displays a fluxogram that summarizes the methodology of this study.

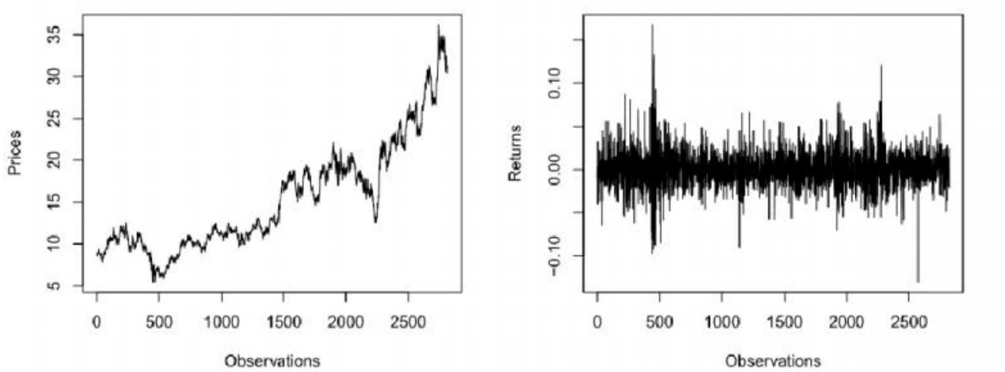


Source: Created by the authors using Adobe Illustrator®.

Figure 2. Methodology fluxogram

RESULTS

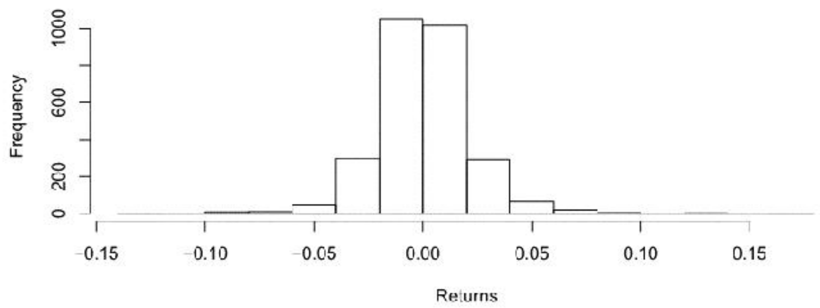
Exploratory analysis: Figure 3 presents graphical representations of BBDC3 stock prices and returns. In Panel of Figure 3a it is possible to notice that the prices seem to be non-stationary, whereas in Panel of Figure 3b the returns have stable variance and are apparently stationary with zero mean.



Source: Created by the authors using R software (R Core Team, 2019).

Figure 3. (a) BBDC3 stock prices and (b) returns in the period between 01/02/2007 and 04/11/2018

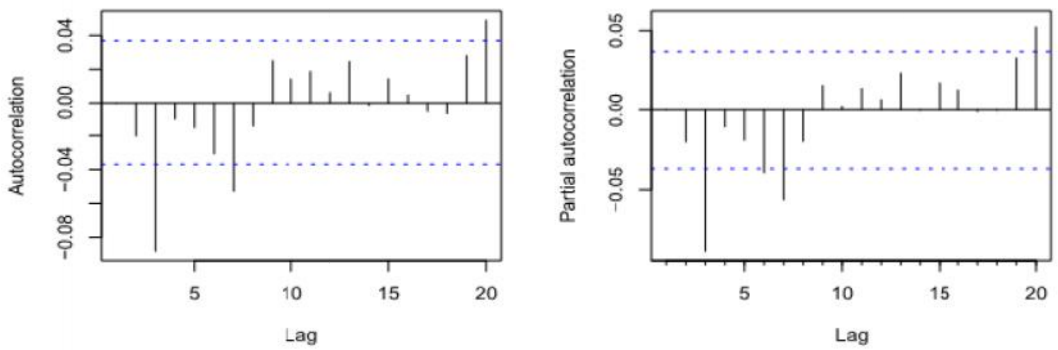
The Figure 4 shows the histogram of BBDC3 returns, in which it is possible to observe that the time series seems to be leptokurtic with approximately symmetrical distribution. This visual analysis is confirmed by the kurtosis coefficient value, given by 7.2219 (kurtosis greater than 3 indicates heavy tails) and by the asymmetry coefficient value, given by 0.3480 (asymmetry shorter than |0.5| indicates symmetry). The arithmetic mean and standard deviation of the BBDC3 returns are 0.0007 and 0.0212, respectively.



Source: Created by the authors using R software (R Core Team, 2019).

Figure 4. Histogram of BBDC3 returns

Stationarity and autocorrelation verification: Figure 5 presents the autocorrelation function (AFC) and partial autocorrelation function (PACF) of BBDC3 returns for the first 20 lags. It is possible to notice that, in both functions, the first three lags have significant autocorrelation, indicating that maybe it is necessary to apply an ARMA model beforehand in order to remove the autocorrelation. However, since this autocorrelation is not persistent, we may assume that the time series is possibly stationary. In order to confirm the evidences found in Figure 5, the ADF and Ljung-Box tests were performed. Both tests resulted in p-value<0.0001, which indicates that the series of returns is possibly stationary and serially autocorrelated. Therefore, the Ljung-Box test indicates that it is necessary to apply an ARMA model in the series of financial returns.



Source: Created by the authors using R software (R Core Team, 2019).

Figure 5 (a). ACF and (b) PACF functions of BBDC3 returns

Application of ARMA model and residual diagnosis: The information criteria and residuals diagnosis of the best five ARMA models, among the 48 models tested, are presented in Table 1.

Table 1. ARMA models applied to BBDC3 returns. LB (r^2): Ljung-Box test p value applied to squared residuals

Model	AIC	BIC	HQIC	LB	LB (r^2)	ALM
ARMA(0,3)	-13619.21	-13589.56	-13616.80	0.1354	<0.0001	<0.0001
ARMA(2,2)	-13.619.44	-13.583.86	-13.614.90	0.1396	<0.0001	<0.0001
ARMA(3,2)	-13.619.70	-13.578.19	-13.613.03	0.3215	<0.0001	<0.0001
ARMA(2,3)	-13.620.03	-13.578.52	-13.613.35	0.3464	<0.0001	<0.0001
ARMA(1,3)	-13620.37	-13584.78	-13615.83	0.2555	<0.0001	<0.0001

Source: Created by the authors.

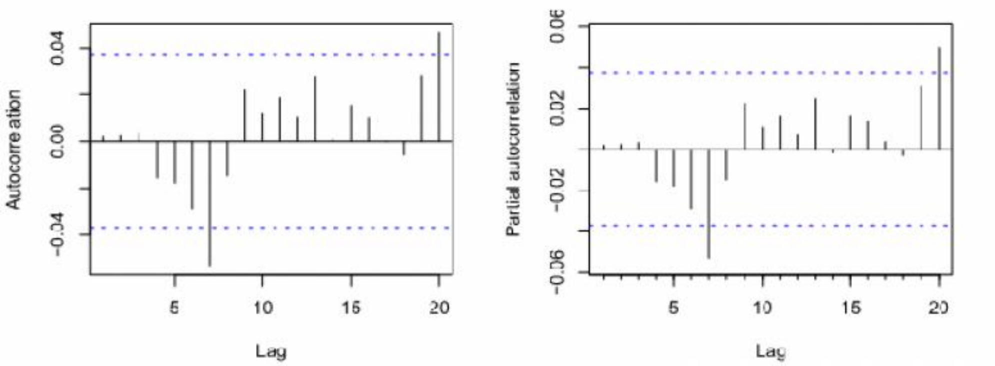
It is possible to notice that ARMA (0,3) model achieved the lowest information criteria and it can remove the significant autocorrelation, according to the Lung-Box test. In the heteroscedasticity verification, the Ljung-Box test applied to the squared residuals and ARCH-LM test indicate that the ARMA (0,3) model residuals are not homoscedastic. Therefore, these results justify the application of a GARCH model.

Table 2. GARCH models applied to BBDC3 returns. LB(r^2): Ljung-Box test p value applied to squared residuals

MODEL	AIC	BIC	HQIC	LB	LB(r^2)	ALM
GARCH(1,1)	-5.0681	-5.0511	-5.0619	0.7278	0.9975	0.9975
GARCH(1,2)	-5.0674	-5.0482	-5.0605	0.7290	0.9977	0.9976
GARCH(2,1)	-5.0674	-5.0482	-5.0605	0.7278	0.9975	0.9975
GARCH(2,2)	-5.0666	-5.0454	-5.0594	0.7282	0.9976	0.9975

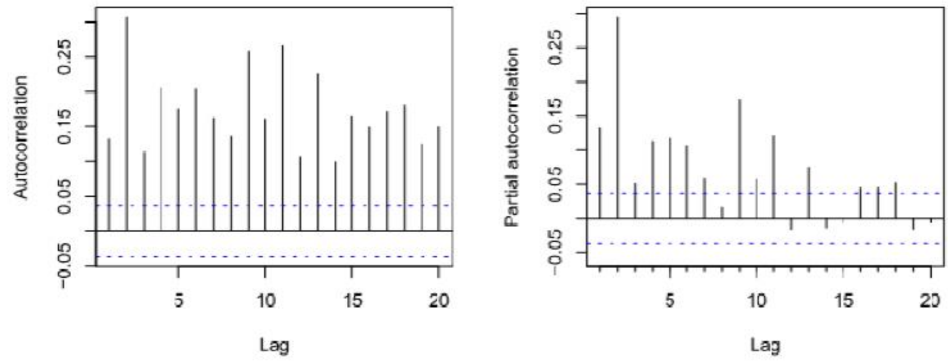
Source: Created by the authors.

In order to confirm the test results, Figures 6 and 7 show the ACF and PACF of residuals and squared residuals from model ARMA (0,3), respectively. Figures 6 and 7 show evidences that the ARMA (0,3) model was able to remove the significant autocorrelation. However, the conditional heteroscedasticity is present in the squared residuals and it persists towards high lag orders. Therefore, Figure 7 indicates that volatility is persistent.



Source: Created by the authors using R software (R Core Team, 2019).

Figure 6. ACF (a) and PACF (b) of ARMA (0,3) residuals.



Source: Created by the authors using R software (R Core Team, 2019).

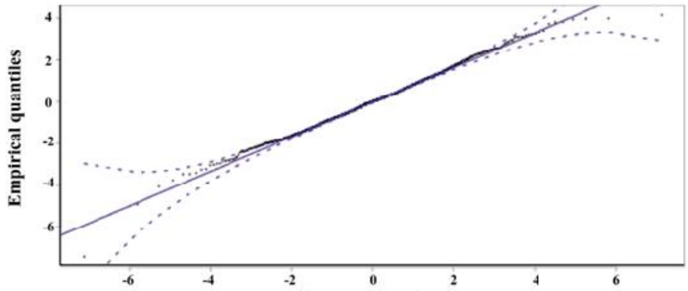
Figure 7. ACF (a) and PACF (b) of ARMA (0,3) squared residuals.

Application of GARCH Models: The application of GARCH(1,1) model resulted in coefficients $\alpha 1 = 0.0615$ and $\beta 1 = 0.9214$, which indicates that volatility is persistent since their sum is close to the unity, agreeing to what was observed in Figure 7a and 7b. Regarding the asymmetric effect, the sign bias test resulted in p-value=0.6766, while the negative sign bias test resulted in p-value=0.0649 and, finally, the positive sign bias test resulted in p-value=0.9100. Considering a 5% significance level, all tests indicate that the leverage effect is not present in the time series under analysis. Finally, Table 2 shows the results of the information criteria and residuals diagnosis from all tested GARCH models. It is possible to notice that the GARCH (1,1) model resulted in the lowest information criteria and is able to remove the significant heteroscedasticity. Figure 8 shows the simulated envelope of GARCH (1,1) residuals. As we may notice, most of the residuals (black dots) lay inside the confidence bands and hence, it is reasonable to assume that they follow a Student's t distribution.

Table 2. GARCH models applied to BBDC3 returns. LB(r^2): Ljung-Box test p value applied to squared residuals

MODEL	AIC	BIC	HQIC	LB	LB(r^2)	ALM
GARCH(1,1)	-5.0681	-5.0511	-5.0619	0.7278	0.9975	0.9975
GARCH(1,2)	-5.0674	-5.0482	-5.0605	0.7290	0.9977	0.9976
GARCH(2,1)	-5.0674	-5.0482	-5.0605	0.7278	0.9975	0.9975
GARCH(2,2)	-5.0666	-5.0454	-5.0594	0.7282	0.9976	0.9975

Source: Created by the authors.



Source: Created by the authors using R software (R Core Team, 2019).

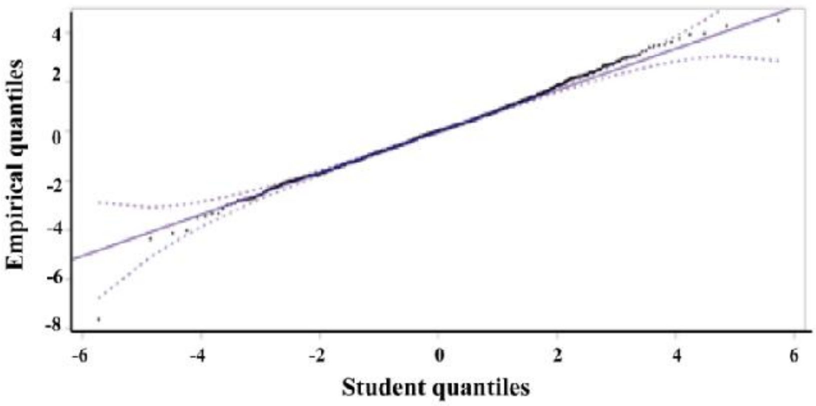
Figure 8 Simulated envelope of GARCH (1,1) standardized residuals

Application of asymmetric GARCH models: As seen before, the sign and size bias tests show evidence that the leverage effect is not present in the BBDC3 series of returns. Regardless, the asymmetric models EGARCH, TGARCH and APGARCH are applied, in order to check if the tests provide useful information about the asymmetry effect. The information criteria and residual diagnosis for these models are presented in Table 3. According to the information criteria, the best models are EGARCH (1,1), APGARCH (1,1) and TGARCH (1,1), being the latter the best model in the fit among those three. In addition, the Ljung-Box and ARCH-LM tests present evidence that all models are able to remove the significant heteroscedasticity. Figure 9 shows the simulated envelope of TGARCH (1,1) squared residuals, where it is possible to observe that most residuals are inside the 95% confidence bands.

Table 3. Information criteria and residual diagnosis for the asymmetric GARCH models. LB(r²): Ljung-Box test p value applied to squared residuals

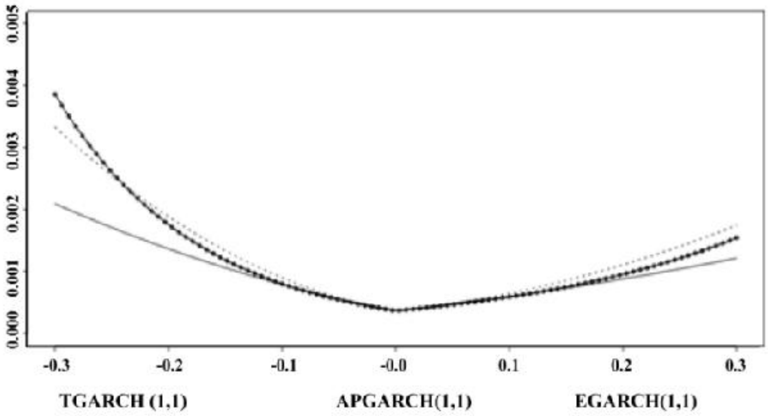
MODEL	AIC	BIC	HQIC	LB(r²)	ALM
EGARCH(1,1)	-5.5897	-5.5727	-5.5836	0.4846	0.5130
EGARCH(2,1)	-5.5888	-5.5676	-5.5812	0.5274	0.5379
EGARCH(1,2)	-5.5890	-5.5699	-5.5821	0.5274	0.5379
EGARCH(2,2)	-5.5881	-5.5648	-5.5797	0.5274	0.5379
TGARCH(1,1)	-5.5911	-5.5741	-5.5850	0.5503	0.7080
TGARCH(2,1)	-5.5905	-5.5713	-5.5836	0.4945	0.7024
TGARCH(1,2)	-5.5897	-5.5685	-5.5820	0.4945	0.7024
TGARCH(2,2)	-5.5890	-5.5657	-5.5806	0.4944	0.7024
APGARCH(1,1)	-5.5911	-5.5720	-5.5842	0.4907	0.5346
APGARCH(2,1)	-5.5904	-5.5692	-5.5827	0.4762	0.5259
APGARCH(1,2)	-5.5891	-5.5657	-5.5806	0.5159	0.5527
APGARCH(2,2)	-5.5894	-5.5639	-5.5802	0.5067	0.5503

Source: Created by the authors.



Source: Created by the authors using R software (R Core Team, 2019).
Figure 9. Simulated envelope of the TGARCH (1,1) standardized residuals.

Finally, Figure 10 shows the news impact curves for EGARCH, TGARCH and APGARCH models. It is possible to notice that the APGARCH (1,1) model, in comparison to TGARCH (1,1) and EGARCH (1,1) models, considers a higher impact of negative returns in volatility. However, as the values of shocks get close to ε_t=-0.3, the EGARCH model starts to give more importance to negative returns than the APGARCH (1,1) model does.



Source: Created by the authors using R software (R Core Team, 2019).

Figure 10. Q-Q plot of TGARCH(1,1) standardized residuals.

Application of GARCH models for persistent volatility: Table 4 shows the information criteria and residual diagnosis for the models IGARCH, FIGARCH and CGARCH applied to the BBDC3 returns.

Table 4. Information criteria and residual diagnosis for the GARCH models for persistent volatility

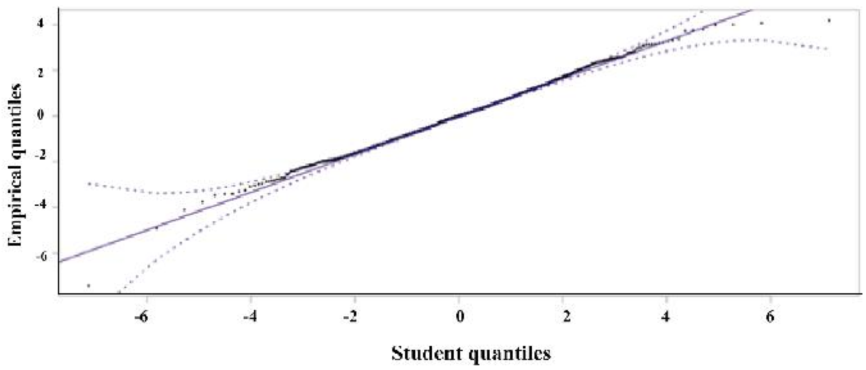
LB(r²): Ljung-Box test p value applied to squared residuals

MODEL	AIC	BIC	HQIC	LB (r²)	ALM
IGARCH(1,1)	-4.5900	-4.5584	-5.5786	0.2467	0.2693
IGARCH(2,1)	-4.5893	-4.5555	-4.5771	0.2467	0.2693
IGARCH(1,2)	-4.5893	-4.5556	-4.5771	0.2546	0.2721
IGARCH(2,2)	-4.5886	-4.5527	-4.5757	0.2597	0.2891
FIGARCH(1,1)	-5,0635	-5,0445	-5,0567	0,9996	0,9996
FIGARCH(1,2)	-4,8297	-4,8086	-4,8221	0,9996	0,9996
FIGARCH(2,1)	-5,0671	-5,0459	-5,0594	0,9988	0,9988
FIGARCH(2,2)	-4,8731	-4,8499	-4,8647	0,9988	0,9988
CGARCH(1,1)	-5,0690	-5,0479	-5,0614	0,9984	0,9984
CGARCH(1,2)	-5,0683	-5,0451	-5,0599	0,9984	0,9984
CGARCH(2,1)	-5,0687	-5,0455	-5,0603	0,9989	0,9988
CGARCH(2,2)	-5,0686	-5,0433	-5,0595	0,9989	0,9988

Source: Created by the authors.

The best models are IGARCH (1,1), FIGARCH (2,1) and CGARCH (1,1). Among these models, the CGARCH (1,1) is the one with lowest information criteria and hence can be considered as the best model. As for the residual diagnosis, the Ljung-Box and ARCH-LM tests show evidences that the heteroscedasticity has been removed.

Figure 11 show the simulated envelope of CGARCH (1,1) squared residuals. Similarly, to the other cases, most residuals are within the 95% confidence bands.



Source: Created by the authors using R software (R Core Team, 2019).

Figure 11. Q-Q plot of CGARCH (1,1) standardized residuals

Forecast and performance comparison: Table 5 presents the forecast results for GARCH (1,1), EGARCH (1,1), TGARCH (1,1), IGARCH (1,1), FIGARCH (2,1) and CGARCH (1,1) models, since those were the best models in the fitting process.

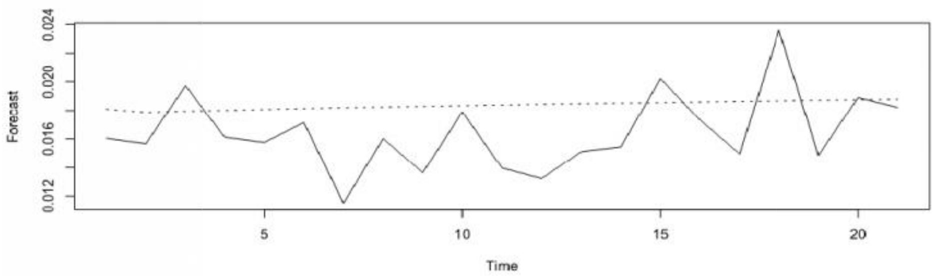
The information criteria for the models are also provided in order to compare the different models and check how the quality of fit affects the forecast.

Table 5. Information criteria and forecast results for the best models in the fit. LB(r²): Ljung-Box test p value applied to squared residuals

MODEL	FIT			FORECAST		
	AIC	BIC	HQIC	MAE	RMSE	MAPE
GARCH(1,1)	-5.0681	-5.0511	-5.0619	0.002815	0.003305	18.7048
EGARCH(1,1)	-5.5897	-5.5727	-5.5836	0.003523	0.004017	23.5418
APGARCH(1,1)	-5.5911	-5.5720	-5.5842	0.003428	0.003909	22.8948
TGARCH(1,1)	-5.5911	-5.5741	-5.5850	0.003548	0.004037	23.7122
CGARCH(1,1)	-5.0690	-5.0479	-5.0614	0.002730	0.003220	18.0967
IGARCH(1,1)	-4.5900	-4.5584	-5.5786	0.003485	0.003953	23.2619
FIGARCH(2,1)	-5.0671	-5.0459	-5.0594	0.003499	0.003966	23.3559

Source: Created by the authors.

According to the information criteria, the asymmetric models are the best in the fit. However, they provide less accurate forecasts than the standard GARCH model and its extensions for persistent volatility. For instance, the TGARCH (1,1) model, although it has the lowest information criteria among all models, results in the biggest forecast errors. The best forecast model is the CGARCH (1,1), followed by the standard GARCH (1,1) model. Forecast results are in agreement with the sign and size bias tests and GARCH (1,1) coefficient values, despite the fact that the asymmetrical models achieved the lowest information criteria among the models of this study. Figure 12 presents the comparison between the forecasted volatility by the CGARCH (1,1) model and the realized volatility computed through intraday data, where it is possible to observe that the model overestimates BBDC3 returns, resulting in forecast values slightly above the central tendency of the original series.



Source: Created by the authors using R software (R Core Team, 2019).

Figure 12. Forecast results from CGARCH (1,1) and realized volatility

CONCLUSION

The goal of this paper was to evaluate and model the leverage effect and persistent volatility of Bradesco Bank stock shares (BBDC3) and to evaluate the performance of the GARCH model and its extensions in the modeling of time series represented by a distribution leptokurtic and symmetric. The evaluation process was made through the sign and size bias tests and the analysis of the coefficients from GARCH (1,1), while the modeling of these characteristics was conducted using the GARCH extensions APGARCH, TGARCH, EGARCH, IGARCH, FIGARCH and CGARCH. According to the sign and size bias tests and the GARCH (1,1) model coefficients, the series of returns under analysis has no significant asymmetry effect, whereas highly persistent volatility is present. Those results apparently reflect on the forecast, since the models for persistent volatility achieved better forecast results than the asymmetric extensions, being the CGARCH (1,1) model the best forecast model in this study, that analyzed a time series symmetric, leptokurtic time series with reasonable variability. This is the main contribution of this paper. However, although the forecast results are worse, the asymmetric models resulted in lower information criteria. Therefore, results indicate that the most common used information criteria might not be a sufficient measurement for comparing different kinds of GARCH models for forecasting. It might be needed to test different penalties in those criteria (e.g. see HOSSAIN *et al.*, 2016). In addition, although a large part of returns time series presents leverage effect, not always the asymmetric models conduct to better results. For this reason, it is necessary to consider other important volatility characteristics, such as the long-range persistence. In BBDC3 returns, for example, the CGARCH and standard GARCH models significantly outperformed the asymmetric extensions. It is possible that the heavy tails of the distribution of returns have influenced these findings. Nevertheless, the CGARCH model slightly overestimates the central tendency, so there is still space for improvement. Future studies shall be conducted considering other models to forecast BBDC3 returns, such as stochastic volatility and multivariate GARCH models.

REFERENCES

Akaike, H. 1973. Information theory and an extension of the maximum likelihood principle. In: B.N. Petrov & F. Csaki Eds.. Proceedings of the 2nd International Symposium on Information Theory. New York, USA: Springer Series in Statistics Perspectives in Statistics, pp. 267-281.

Andersen, T.G., &Bollerslev, T. 1998. Answering the skeptics: Yes, standard volatility models do provide accurate forecasts. International Economic Review, 39, pp. 885-905.

Ané, T. 2006. Na analysis of the flexibility of Asymmetric Power GARCH models. Computational Statistics & Data Analysis, 51, pp. 1293-1311.

Atkinson, A.C. 1985. Plots, Transformations, and Regression: an introduction to graphical methods of diagnostic regression analysis, Clarendon Press, Oxford, UK.

Baillie, R.T., Bollerslev, T., Mikkelsen, H.O. 1996. Fractionally integrated generalized autoregressive conditional heteroskedasticity. Journal of Econometrics, 74, pp. 3-30.

Beg, A.B.M.R.A., Anwar, S. 2014. Detecting volatility persistence in GARCH models in the presence of the leverage effect. Quantitative Finance, 1412, pp. 2205-2213.

Black, F. 1976. Studies in stock price volatility changes. Proceedings of the 1976 Meeting of the Business and Economic Statistics Section;American Statistical Association,Washington DC, pp. 177-181.

Bollerslev, T. 1986. Generalized autoregressive conditional heteroskedasticity. Journal of Econometrics, 313, pp. 307-327.

Box, G.E.P., Jenkins, G. 1970. Time Series Analysis, Forecasting and Control,Holden-Day,San Francisco, USA.

Chou, R.Y. 1988. Volatility persistence and stock valuations: some empirical evidence using GARCH. Journal of Applied Econometrics, 3, pp. 279-294.

Christie, A.A. 1982. The stochastic behavior of common stock variances: value, leverage and interest rate effects. Journal of Financial Economics, 10, pp. 407-432.

Dickey, D.A., Fuller, W.A. 1981. Likelihood ratio statistics for autoregressive time series with a unit root. Econometrica, 494, pp. 1057-1073.

DIEESE. 2020. Relatório dos bancosem 2019. DepartamentoIntersindical de Estatísticas e EstudosSocioeconômicos. URL https://www.dieese.org.br/desempenhodosbancos/2020/desempenhoDosBancos2020.html. Acessoem: nov 2020.

Ding, Z., Granger, W.K., Engle, R.F. 1993. A long memory property of stock market returns and a new model. Journal of Empirical Finance, 1, pp. 83-106.

Engle, R.F. 1982. Autoregressive conditional heteroscedasticity with estimates of the variance of United Kingdom inflations. Econometrica, 504, pp. 987-1007.

Engle, R.F., Bollerslev, T. 1986. Modelling the persistence of conditional variances. Econometric Reviews, 94, pp. 405-420.

Engle, R.F., Lee, G. 1999. A permanent and transitory component model of stock return volatility. In: R.F. Engle & H. White Eds.. Cointegration, Causality and Forecasting: A Festschrift in Honor of Clive W. J Granger. Oxford, UK: Oxford University Press, pp. 475-497.

Engle, R.F., Mustafa, C. 1992. Implied ARCH models from options prices. Journal of Econometrics, 521-2, pp. 289-311.

Engle, R.F., Ng, V.K. 1993. Measuring and testing the impact of news on volatility. The Journal of Finance, 485, pp. 1749-1778.

Engle, R.F., Patton, J.A. 2001. What good is a volatility model? Quantitative Finance, 1, pp. 237-245.

Fama, E.F. 1965. The behavior of stock-market prices. Business, 38, pp. 34-105.

Glosten, L.R., Jaganathan, R., Runkle, D. 1993. On the relation between the expected value and the volatility of the normal excess return on stocks. Journal of Finance, 48, pp. 1779-1801.
Hannan, E.J., Quinn, B.G. 1979. The determination of the order of an autoregression. Journal of the Royal Statistical Society, Series B. Methodological, 412, pp. 190-915.
Hansen, P.R., Lunde, A. 2005. A forecast comparison of volatility models: does anything beat a GARCH1,1? Journal of Applied Econometrics, 207, pp. 873-889.
Higgins, M.L., Bera, A.K. 1992. A class of nonlinear ARCH models. International Economic Review, 331, pp. 137-158.
Hossain, A., Rigby, R., Stasinopoulos, M., Enea, M. 2016. Centile estimation for a proportion response variable. Statistics in Medicine, 356, pp. 895-904.
Infomoney.2018. Financial Stocks Database, n.d. URL https://www.infomoney.com.br/mercados/ferramentas/historico-de-cotacoes.
Lamourex, C.G., Lastrapes, W. 1993. Forecasting stock-return variance: toward an understanding of stochastic implied volatilities. The Review of Financial Studies, 62, pp. 293-326.
Lanne, M., Saikkonen, P. 2005. Non-linear GARCH models for highly persistent volatility. The Econometrics Journal, 82, pp. 251-276.
Lee, G., Engle, R.F. 1999. A permanent and transitory component model of stock return volatility. cointegration, causality and forecasting: A festschrift in honor of Clive W.J. Granger. Oxford University Press, New York, USA.
Ljung, G.M., Box, G.E.P. 1978. On a measure of lack of fit in time series models. Biometrika, 652, pp. 297-303.
Mandelbrot, B. 1963. The variation of certain speculative prices. Business, 36, pp. 394-419.
Morettin, P.A. 2011. EconometriaFinanceira: Um cursoemsériestemporaisfinanceiras 2a ed.. Blucher, Brasil.
Nelson, D.B. 1991. Conditional Heteroscedasticity in asset returns. Econometrica, 592, pp. 347-370.
Perobelli, F., Reis, D., Mattos, R., Corrêa, W. 2013. Options pricing for Petrobras share: A comparison between realized volatility and conditional heteroskedastic estimations. In: First International Workshop in Financial Econometrics. Natal, RN, Brazil, 2013. URL https://sites.google.com/site/workshopnatal2013/program.
Poon, S., Granger, C.W.J. 2003. Forecasting Volatility in Financial Markets: A Review. Journal of Economic Literature, 41, pp. 478-539.
R Core Team. 2019. A Language and Environment for Statistical Computing. R Foundation for Statistical Computing, Austria, 2019. URL https://www.R-project.org.
Robinson, J., Glean, A., Moore, W. 2018 How does news impact on the stock prices of green firms in emerging markets? Research in International Business and Finance. 45, pp. 446-453.
Schwarz, G. 1978. Estimating the dimension of a model. The Annals of Statistics, 62, pp. 461-464.
Schwert, G.W. 1989. Why does stock market volatility change over time? Journal of Finance, 44, pp. 1115-1153.
Taylor, S.J. 1986. Modelling financial time series. John Wiley & Sons, England, United Kingdom.
Trindade, M. A. S., Floquet, S., Silva Filho, L. M. 2020 Portfolio theory, information theory and Tsallis statistics. Physica A: Statistical Mechanics and its Applications, 541. Available online at https://www.sciencedirect.com/science/article/abs/pii/S0378437119318370.1
Tsay, R.S. 2005. Analysis of Financial Time Series: Financial Econometrics. Wiley-Interscience Publication, John Wiley & Sons, Unite States of America.
Zakoian, J.M. 1994. Threshold heteroskedastic models. Journal of Economic Dynamic and Control, 18, pp. 931-944.

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