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# ANALYSIS AND CONJECTURES REGARDING THE PRIME GAPS USING C# PROGRAMMING LANGUAGE

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### ABSTRACT

This paper brings four conjectures, based on analysis of prime gaps, to the knowledge of the scientific community. This research and its results were based on data obtained directly by computational algorithms, which were developed by the authors of this paper, for the analysis of prime numbers and prime gaps. These data were the basis for the conjectures proposed here. However, they are only preliminary results and conjectures that require more investigation and deeper analysis for future proof or refutation. The first conjecture presented states that the graph for the first occurrences of all prime gaps behaves according to an exponential function that tends to a quadratic function. The second proposes which are the only prime gaps that occur consecutively in the sequence of prime gaps. The third conjecture proposes the creation of a new sequence of intervals, but now between prime gaps, in which the sum of its infinite elements will be equal to zero and consequently, may serve as a feasible argument that the twin primes occur infinitely.



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# INTRODUCTION

The research of prime numbers has great relevance not only in mathematics, but in several fields of knowledge (NEUKIRCH, 2013). Given its importance, several means and tools were created throughout history for its research. One of these ways of studying and looking for patterns in prime numbers is a sequence of numbers of the intervals between prime numbers, also known as prime gaps. These numbers were already known and studied since antiquity (Stillwell, 2010). Prime gaps, or PGs as it will be called from now on, are a sequence of numbers that can express important properties about the behavior of prime numbers. Further on, the main initial concepts regarding this sequence will be detailed. For now, it must be said that any tool or equation that makes it possible to predict the sequence of PGs will also make it feasible to determine the prime numbers (Granville, 1995a). However, although this result is far from being achieved, there are other properties that can be extracted and studied based on the sequence of PGs that would be of value for the study of the prime numbers. In this paper, therefore, some of these properties will be studied and, based on data obtained with computational algorithms, preliminary conjectures will be proposed, which in the future should be revised, refined, better analyzed, so that they can finally be proved or disproved.

The research of the behavior of the function that describes the first occurrence of each PG will be the first point to be analyzed. The second will deal specifically with PG numbers that occur consecutively, for example, ([…], 6, 6, [...]) etc. Then, based on the PG sequence, a new sequence will be derived with the interval between PGs, which will be called IPG, that will be analysed and another conjecture will be proposed concerning its behavior. For this entire research, it is necessary to emphasize, the main data that support it were obtained through algorithms, developed by the authors themselves, both for the analysis of the primes, as well as the PGs and IPG etc., which require more deep analysis. The readers are invited to review these codes, which follow below, to replicate their results, to increase the quantity of data, to better test each of the conjectures presented here and / or to present a better demonstration of the behavior of such sequences of numbers.

# MATERIALS AND METHODS

**Algorithms and programing language used in this research:** Using the C# programming language, the following algorithms and methods were developed for this research of PGs. Although other programming languages may be more suited to improve the performance of the analysis, the authors were limited to use a language that they had better familiarity and qualification to program, this being C#. However, such algorithms can easily be replicated and translated into other programming languages by the reader. Below are the main methods and algorithms used:

Algorithm (1) to check if the input number is a prime number:

```
public static bool CheckIfPrime(ulong number)
    {
        ulong counter = 0;
        double test1 = (number + 1) / 6;
        double test2 = (number - 1) / 6;

        if (number < 1) return false;

        if (number == 2) return true;

        if (number > 2 && number % 2 == 0 || number > 3 && number % 3 == 0 || number
                            > 5 && number % 5 == 0 || number > 7 && number % 7
                            == 0 || number > 11 && number % 11 == 0 || number > 13
                            && number % 13 == 0 || number > 17 && number % 17
                            == 0 || number > 19 && number % 19 == 0 || number > 23
                            && number % 23 == 0 || number > 29 && number % 29
                            == 0 || number > 31 && number % 31 == 0) return false;
        else if (test1 % 1 != 0 | test2 % 1 != 0) return false;
        else
        {
            for (ulong i = 1; i <= Math.Sqrt(number); i += 2) if (number % i == 0)
                            counter++;

            if (counter == 1) return true;
            else return false;
        }
    }
```

Algorithm (2) that checks what will be the next prime number in relation to the number indicated in the input:

```
public static ulong NextPrime(ulong number)
    {
        bool result;
        do
        {
            number++;
            result = CheckIfPrime(number);
        } while (!result);

        return number;
    }
```

Algorithm (3) that indicates the number of prime numbers in the defined range:

```
public static ulong QtyPrimesInInterval(ulong verificationStart, ulong verificationEnd)
    {
        ulong interval = verificationEnd - verificationStart;
        ulong QtyPrimes = 0;

        do
        {
            if (CheckIfPrime(verificationStart))
                QtyPrimes++;

            verificationStart++;
        } while (verificationStart != verificationEnd + 1);

        return QtyPrimes;
    }
```

Algorithm (4) that returns the prime gaps of an interval defined with an ulong vector:

```
public static ulong[] PrimeGaps(ulong verificationStart, ulong verificationEnd)
    {
        ulong[] valores = new ulong[QtyPrimesInInterval(verificationStart,
verificationEnd)];

        ulong starting = NextPrime(verificationStart);
        ulong next = NextPrime(starting);
        ulong interval = next - starting;
        ulong i = 1;
        valores[0] = interval;

        while (next < verificationEnd)
        {
            starting = next;
            next = NextPrime(starting);
            interval = next - starting;
            valores[i] = interval;
            i++;
        }
        return valores;
    }
```

Algorithm (5) that checks if there is a number in the list of prime gaps that occurs consecutively:

```
public static ulong FindNumberConsecutivePrimeGap(ulong primeGap)
    {
        ulong verificationStart = 0;
        ulong count = 0;
        ulong interval;
        ulong intervalAnt=0;
        ulong starting;
        ulong next;
        bool result=true;

        do
        {
            starting = NextPrime(verificationStart);
            next = NextPrime(starting);
            verificationStart = next - 1;
            count++;
            interval = next - starting;

            if (intervalAnt==primeGap&&interval==primeGap)
            {
                result = false;
            }
            else
            {
                intervalAnt = interval;
                Console.Write(count + ", ");
            }

        } while (result);

        return count;
    }
```

Algorithm (6) indicates the distance (quantity of numbers leading up) between the beginning of the sequence of prime gaps and the first occurrence of a chosen value:

```
public static ulong NumbersUntilPrimeGap(ulong primeGap)
    {
        ulong verificationStart = 0;
        ulong count = 0;
        ulong interval;
        ulong starting;
        ulong next;

        do
        {
            starting = NextPrime(verificationStart);
            next = NextPrime(starting);
            verificationStart = next-1;
            count++;
            interval = next - starting;
        } while (interval != primeGap);

        return count-1;
    }
```

Algorithm (7) returns a list with the occurrence distances from the chosen prime gap to the prime gap 2:

```
public static List<string> ListOfDistanceOfEachPrimeGapNumber(ulong ResultPrimeGap)
    {
        List<string> lista = new List<string>();

        if (ResultPrimeGap % 2 != 0)
        {
            MessageBox.Show("Prime numbers must not be even numbers.");
        }
        else
        {
            for (ulong i = 2; i <= ResultPrimeGap; i+=2)
            {
                lista.Add(Convert.ToString(i) + ", " +
                                Convert.ToString(NumbersUntilPrimeGap(i)));
            }
        }
        return lista;
    }
```

Algorithm (8) returns a list with only the distance between the chosen prime gaps.

```
public static long[] DistanceBetwPrimeGaps(long primeGapnumber, ulong
                            verificationStart, ulong verificationEnd)
    {
        long verStart = Convert.ToInt64(verificationStart);
        long verEnd = Convert.ToInt64(verificationEnd);

        long[] primeGaps = PrimeGapsArrayLong(verStart, verEnd);

        long QtyPrimes = QtyPrimesInIntervalLong(verStart, verEnd);

        long QtyRepetition = QtyRepetitionArray(primeGaps, primeGapnumber);

        long[] OnlyDistance = new long[QtyRepetition];

        long count = -1;
        int position = 0;

        for (long i = 0; i < QtyPrimes; i++)
        {
            count++;
            if (primeGaps[i] == primeGapNumber)
            {
                OnlyDistance[position] = count;
                count = -1;
                position++;
                if (position == QtyRepetition)
                {
                    break;
                }
            }
        }

        return OnlyDistance;
    }
```

**Table I: DistPG (y), in quantity of predecessor elements, of the referred prime gaps (x).**

| | | | | | | | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| *x* | 2 | 4 | 6 | 8 | 10 | 12 | 14 | 16 | 18 | 20 | 22 | 24 | 26 | 28 | 30 | 32 | 34 |
| *y* | 1 | 3 | 8 | 23 | 33 | 45 | 29 | 281 | 98 | 153 | 188 | 262 | 366 | 428 | 589 | 737 | 216 |
| *x* | 36 | 38 | 40 | 42 | 44 | 46 | 48 | 50 | 52 | 54 | 56 | 58 | 60 | 62 | 64 | | |
| *y* | 1182 | 3301 | 2190 | 1878 | 1830 | 7969 | 3076 | 3426 | 2224 | 3792 | 8027 | 4611 | 4521 | 3643 | 8687 | | |
| *x* | 66 | 68 | 70 | 72 | 74 | 76 | 78 | 80 | 82 | 84 | 86 | 88 | 90 | | | | |
| *y* | 14861 | 12541 | 15782 | 3384 | 34201 | 19025 | 17005 | 44772 | 23282 | 38589 | 14356 | 44902 | 34214 | | | | |

**Initial Concepts:** Prime gaps are a sequence of numbers obtained through the difference between two consecutive prime numbers, such that $P\ \ = p_{n+1} - p_n$, (where p is a prime number and PG is a prime gap), which results, for example, in $P_{\ 1} = 1, P_{\ 2} = P_{\ 3} = 2, P_{\ 4} = 4$ etc. (WEISSTEIN, 2001). Thus, using the algorithm (4), for the following primes, the first 60 PGs are defined, in parentheses, as an example:

|2, (1), 3|, |3, (2), 5|, |5, (2), 7| , |7, (4), 11| , |11, (2), 13| , |13, (4), 17| , |17, (2), 19| , |19, (4), 23| , |23, (6), 29| , |29, (2), 31| , |31, (6), 37| , |37, (4), 41| , |41, (2), 43| , |43, (4), 47| , |47, (6), 53| , |53, (6), 59| , |59, (2), 61| , |61, (6), 67| , |67, (4), 71| , |71, (2), 73| , |73, (6), 79| , |79, (4), 83| , |83, (6), 89| , |89, (8), 97| , |97, (4), 101| , |101, (2), 103| , |103, (4), 107| , |107, (2), 109| , |109, (4), 113| , |113, (14), 127| , |127, (4), 131| , |131, (6), 137| , |137, (2), 139| , |139, (10), 149| , |149, (2), 151| , |151, (6), 157| , |157, (6), 163| , |163, (4), 167| , |167, (6), 173| , |173, (6), 179| , |179, (2), 181| , |181, (10), 191| , |191, (2), 193| , |193, (4), 197| , |197, (2), 199| , |199, (12), 211| , |211, (12), 223| , |223, (4), 227| , |227, (2), 229| , |229, (4), 233| , |233, (6), 239| , |239, (2), 241| , |241, (10), 251| , |251, (6), 257| , |257, (6), 263| , |263, (6), 269| , |269, (2), 271| , |271, (6), 277| , |277, (4), 281| , |281, (2), 283|.

Briefly: 1, 2, 2, 4, 2, 4, 2, 4, 6, 2, 6, 4, 2, 4, 6, 6, 2, 6, 4, 2, 6, 4, 6, 8, 4, 2, 4, 2, 4, 14, 4, 6, 2, 10, 2, 6, 6, 4, 6, 6, 2, 10, 2, 4, 2, 12, 12, 4, 2, 4, 6, 2, 10, 6, 6, 6, 2, 6, 4, 2.

Due to the importance of prime numbers, several methods have been created since antiquity to analyze existing patterns regarding these numbers in order to propose explanations and conjectures for the understanding their behavior. PGs emerged with this main purpose, evolving over time to become an important subject with diverse and profound properties as well as important conjectures (NAZARDONYAVI, 2012). Some of these properties are, for example: PG 1 occurs only once between the primes 2 and 3, while 2 being the only prime number that is even. Consequently, all subsequent PGs are even numbers (WEISSTEIN, 2001). There is only one consecutive pair of PGs 2, (ie: 2, 2), which occurs between the primes 3, 5 and 7, because 5 is the only prime number ending in 5 (WOLF, 2001). There is a conjecture, yet to be proven, that the PG 2, also known as Twin Primes, occurs endlessly. It was demonstrated by the mathematician Yitang Zhang that there are infinite PGs with an interval less than or equal to $7 * 10^7$. Subsequent research, by the Polymath project, managed to extend its results to 246 (ZHANG, 2014; POLYMATH, 2014).

# DISCUSSION AND RESULTS

**The distance between the origin and the occurrence of a Prime Gap (DistPG):** Checking the sequence of PGs and their first occurrence in the sequence, (which will be called here as DistPG), and using the algorithm (7), it was possible to construct the following table I, in which x indicates the PG and y is the position of its first occurrence in sequence of PGs, DistPG: Note that, in table I, the occurrence of each PG does not follow a constant increasing pattern, but each PG can have much smaller or larger intervals in relation to its neighbors. For example, we see that the PG 72 occurs after 3384 positions in the sequence, however, the PG 70 occurs after 15,782 and the PG 74 after 34,201 positions. Thus, it appears that some specific PGs occur more frequently than others.

In this way, if the terms y are placed in ascending order, and the values of x are changed by 1 to 45, the graph in Figure 1 is formed:

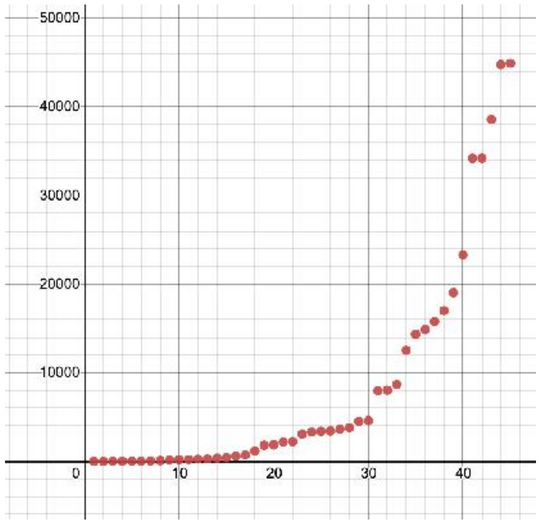


**Figure 1. Points in Table 1 plotted in a graph, with increasing values**

Looking at the graph above, it is noticeable a plausible exponential or quadratic function pattern forming. By analyzing the data, it is tenable to raise some assumptions:

**Conjecture (1):** The function that describes the growth of the interval between primes, DistPG, follows an exponential pattern that tends to quadratic, by the following approximation: [1] $y = b^x + x^2 + 3x$**,** for $x > 0$, considering $b \rightarrow 1$, such that the disparities in some of its intervals will be reduced as larger PGs are computed, tending towards a quadratic function. Taking the above conjecture and placing b = 1.12 as an example, the following graph is obtained:

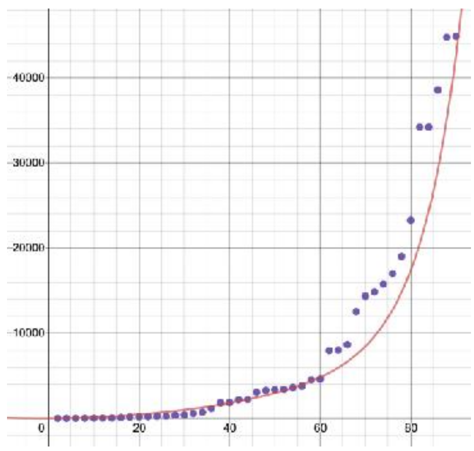


**Figure 2. Graph of the function [1], in red, with the points of Table 1, in purple**

It must be made clear that the conjecture stated above and the proposed equation are only an approximation and do not reflect exact values.

However, the data collected allowed us to demonstrate the likely behavior of the DistPG distribution function. The reader is invited to extend this research and its results.

**The consecutive repetition of Prime Gap numbers:** It was also feasible to verify, by analyzing the data obtained with the algorithm (5) and analytically with respect to the PG 2, (which have no consecutive repetitions, aside between 3, 5 and 7, as seen in the introduction) (WOLF, 2001), that, unlike PG 2, there are PGs that repeat consecutively in the sequence of PGs. Some of the examples found were numbers 6, 12, 18, 24, 30, 36 etc. For example, PG 6 is consecutive, (6, 6), for the first time in positions 15-16. PG 12 is consecutive, (12, 12), in positions 46-47. Likewise, the PG 18 is consecutive in positions 2284-2285 and PG 24 in positions 1938-1939, PG 30 in positions 6905-6906, PG 36 in positions 22,506-22,507, PG 42 in positions 21,805-21,806, PG 48 in positions 254,478-254,479, PG 54 in positions 432,986-432,987, PG 60 in positions 342,724-342,725 etc. However, the PGs between these numbers were analyzed up to position 10,000,000 of the sequence and no consecutive occurrences were found, except those that follow this pattern. With these data, it is tenable to suppose a probable logic for PG numbers that occur consecutively:

**Conjecture (2):** Every prime gap such that [2] $P_n = 6 + (n-1) * 6$, and $n \in N^*$ , will have consecutive terms in the PGs sequence. No number other than these, except for PG 2, will be repeated consecutively at least once. For the above conjecture, the data obtained with the algorithm (8) were used. Having searched the first 10,000,000 terms of the PGs, only the numbers below PG 96 (this one in positions 3,059,821-3,059,822 ) were found in this sequence. More terms of the PG must be verified to find more results. Therefore, the reader is again invited to assist in obtaining more data to support or refute the above conjecture.

**The interval between Prime Gaps (IPG):** Another way to study the prime numbers can be developed not only with the study of the interval between them, but also with the interval between the interval of primes, that is, the interval between prime gaps. For this new sequence the acronym IPG will be used.

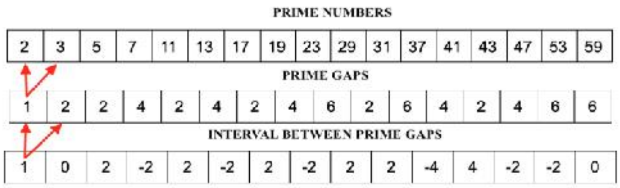


**Figure 3. Tables representing three layers, where the first represents prime numbers, followed by their respective prime gaps, followed by the interval between prime gaps**

Thus, three layers of sequence numbers are now formed, Figure 3, the first layer being the primes, the second is the PGs and the third is the IPG: Note that with this new layer, if the first element is disregarded, (1), the sum of all the elements in the IPG tends to 0. That is, whenever positive elements appear, there will be equivalent negative elements later on that will annul each other out. Thus, even if at some point the sum of the elements is not 0, continuing on the sum of the next numbers, the sum will be 0. Therefore, it is observed here the existence of cycles in the PGs such that the PG 2, which does not occur consecutively except once (WOLF, 2001), must recur. Therefore, it is feasible to conclude that if there is a consecutive pair of primes with PG greater than 2, there will inevitably be another pair of consecutive prime numbers with PG 2 that follows in the future pairs. And this one, with PG 2, will necessarily follow another prime pair that, in relation with the previous one, will *not* have PG 2.


**Figure 4. The sum of the IPG terms, except for the first term 1, is equal to 0**

Analyzing Figure 4, and having in mind the explanation above, the sum of the n IPG terms, expressed below in [3], will be 0:

$$[3]\sum_{n-2}^{\infty} II_n = 0$$

That is, the existing PGs will always converge to the PG 2. Therefore, it will be useful to have a sequence with the interval between the occurrence of the PG 2, in the sequence of PGs. This new sequence of intervals between the PG 2 in the PG sequence, (which will be called ITP2), have the following 102 first elements: 0, 1, 1, 2, 2, 3, 2, 5, 1, 4, 1, 5, 1, 1, 3, 2, 4, 2, 3, 4, 11, 1, 5, 8, 5, 4, 3, 2, 3, 19, 1, 1, 3, 3, 18, 1, 2, 1, 3, 7, 10, 4, 2, 2, 2, 9, 4, 3, 1, 16, 2, 5, 2, 2, 8, 8, 1, 5, 1, 5, 4, 5, 1, 2, 1, 2, 8, 3, 6, 2, 6, 19, 3, 6, 5, 4, 2, 6, 2, 19, 1, 13, 3, 9, 1, 2, 5, 3, 1, 1, 6, 1, 5, 2, 2, 2, 10, 11, 5, 3, 7, 2.[1] To understand it, let us make an analysis of its first elements: Between the first occurrence and the second one of the PG 2, there was no interval, so the result is 0, since the numbers were consecutive. In the second element, we have 1, since there was a number between the PGs 2, (eg: 2, X, 2 - where X is a PG number not explicit here), and so on. An adimitable conclusion, therefore, is that even if there are large intervals between the PGs 2, they will always converge again to 2, as already stated. Hene, it is tenable to conjecture, based on [3], that:

**Conjecture (3):** Tending toward infinity, the “last” PG number should be 2. Consequently, the twin primes occur infinitely in the sequence of prime numbers, since it is proven that they are infinite, (FURSTENBERG, 1955). It should be noted that, even though there is already a famous conjecture for the infinite occurrence of twin primes, (DE POLIGNAC, 1849; NAZARDONYAVI, 2012), the one presented here is now based on the unproven [3] of the IPGs. That is, if this conjecture is true, consequently twin primes occur infinitely.

# CONCLUSION

In this research, four conjectures were presented based on preliminary data from the analysis of PGs. By the analysis of these data, obtained with the help of the referenced algorithms, it was feasible to propose these hypotheses that seek a better understanding of the prime numbers and the prime gaps. The first conjecture approximates the probable function for the behavior of the distances of the first occurrence of each PG, DistPG. The second conjecture proposes the sequence of PG numbers that will occur consecutively at least once. The third conjecture is based on the construction of a new sequence of numbers, called the interval between prime gaps (IPG), and based on the data obtained, conjecture that the sum of all its elements except the first is equal to 0. Consequently, it states that there are infinite prime twins. As the conjectures presented here are still based on limited data, therefore, there may be a considerable reduction in their reliability. This was due mainly to the difficulty in obtaining more data for this research, which required better equipment and algorithms for data analysis. Even so, the data obtained and the conjectures proposed can form an initial basis for future more deep research.

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[1]Sum 441. Standard deviation: 4,139071893. Arithmetic average: 4,323529412. Mean: 3. Mode: 2.

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