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EFFECTS OF USING CUTTER WITH ASYMMETRIC TIP RADIUS ON ROOT STRESS OF THIN AND SOLID RIMMED INTERNAL GEAR

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ABSTRACT

Nowadays internal spur gears are used in many industrial areas because of their numerous advantages such as reduced sliding velocity and wear, higher transmission ratio, higher running efficiency. In contrast of the external gear an internal gear is generated by pinion cutters towards inside of gear blank. In this study, bending stress of internal spur gear generated with the cutter that has asymmetric tip radius was investigated. This condition is let to use larger tip radius on one side. Firstly, a mathematical equation of pinion cutter is derived and then the points of internal gear are obtained by using coordinate transformation, differential geometry and gearing theory in MATLAB. For realizing the 2D design the points of internal gears are exported to CATIA program. Then for determining the relation between tip radius and bending stress, the case studies are conducted respectively with ANSYS program. According to preliminary results, using asymmetric trochoid concept is rather advantageous when comparing symmetric one.

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INTRODUCTION

Thin rimmed internal gears could be used in applications where lightweighting is demanded. It is well known fact that using thin rim are weakened the internal gear tooth and increased root stresses which result in tooth breakage. Trochoid geometry is the predominant factor on root stress. With increasing pinion cutter tip radius, the trochoid geometry is getting smoother and improved in view of strength. Using asymmetric tip radius in pinion cutter allows getting larger tip radius values in tensile side which is critical since the crack begins from there. In the literature the mathematical expressions of external and internal gears are have been revealed by the researchers for decades. These studies also include principles of manufacturing and assembly in detail beside sizing and designing. (Buckingham 1949; Colbourne, 1987; Colbourne, 1990; Litvin, 1994; Radzevich & Darle, 1994; Buckingham & Buckingham, 1999, Jelaska; 2012). Yang (2007) obtained the mathematical expressions of involute asymmetric internal spur gears and helical gears with two coordinate transformations. First, the pinion set that will manufacture the internal gear is expressed with an imaginary rack – cutter set and then the equations of internal gears are derived. He also conducted a contact analysis of teeth to investigate the mistakes that resulting from the assembly. The effects of the asymmetry on stress are investigated with finite element method and then the prototype manufacturing of the designed gears is conducted with rapid prototyping method. Fetvaci (2016) determined in his study that the equations used for mathematical modeling of internal and external gears which reveals by Yang (2007) are not fully achieve the addendum height. This situation led to the need for additional operations in the two-dimensional design of gears. For solving these problems the equations of line of action have been used and the exact addendum height is determined. Muni and Muthuveerappan (2009) investigated the internal asymmetric spur gear design with the conventional and direct design methods and reveal various equations.

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They showed the undercutting and tip interference boundaries of asymmetric internal gears with diagrams. They examined the effects of different parameters on stress with finite element method for finding the optimum mechanism. In order to maximize the asymmetry, it was stated that the side with the highest possible contact ratio should be loaded at the smallest possible addendum height. Kawalec and Wictor (2004) examined the stress that occurs on internal gears with an analytical method. They proposed a new model because of the current standards are insufficient and depends on many acceptations. They also validated the proposed method with finite element method and indicated that the tension is mostly dependent on the tip radius of the pinion type cutting tool and the number of teeth and that the maximum tension depends on the pinion type cutting tool tip radius and the number of teeth of the internal gear. Sanchez et al. (2016) developed a load sharing model for calculating the contact and root stresses that occur on internal gear mechanisms. For this, they based the minimum elastic potential criteria. It has been revealed that the contact points, except addendum height, little affected with the gear parameters as a significant result. The obtained stress values are verified with finite element method. Oda et al. (1984) investigated the effect of internal gear rim thickness on tooth root stress and fatigue strength of internal teeth with using two dimensional finite element method. According to this it is seen that the stress values increase with the increasing of rim thickness. It has been found that when the rim thickness is smaller than 3 times the modulus, fatigue strength is decreased in other cases it is revealed that the rim thickness affected very less. Then the study has been verified with bending fatigue test.

Oda and Miyachika (1986) examined the root stresses of the internal gears for different number of teeth and profile shifting with the elasticity theory. They determined the stress concentration factor and the point that maximum stress. A simple empirical expression was developed from the obtained results. It is seen that the results that obtained from the analysis of finite element are consistent with the obtained result. Litvin et al. (1994) set the tool parameters that necessary for not having undercutting in their work. They stated the limit conditions in order to avoid tip interference for different assembly conditions. They showed these expressions with graphics and tables. According to this it has been stated that the number of the selected tool and internal gear teeth should be 32 to 200 and that of the number of teeth should be smaller than 9,162 minor than 1,004 times of the number of internal gear teeth. Karpat et al. (2014) investigated the effect of internal gears on tooth stiffness and stress in different rim thicknesses and shapes. They used finite element method for this study. It is seen that the stiffness and stress values are decreased with increasing rim thickness. Karpat et al (2015) studied the mathematical model of the internal gears with the double coordinate transformation. The mathematical model was written in the computer program and points that create the gear were sent to the design program. The internal gear with the two-dimensional different rim thickness, shape and the tip radii were subjected to finite element analysis. As result, it is obtained that the rim shape has a significant effect on stress values.

Yilmaz et al. (2017) created the mathematical models of internal gears that made of pinion sets for asymmetric profile. By the computer program the points of the internal gear were obtained and the two dimensional design was carried out in the CATIA program. The involute asymmetric and asymmetric trochoid internal gear models were subjected to finite element analysis. In this study, asymmetric trochoid concept is offered to decrease root stresses of internal spur gear teeth with thin rim. For this first the pinion cutter which generates internal gear is determined. Then the internal gear is constituted with using kinematic relation between cutter and internal gear, and gearing theory in MATLAB program. 3D design is created with CATIA. Stress analyses are conducted in ANSYS with symmetric and asymmetric tip radius and rim thickness.

MATERIALS AND METHODS

Pinion cutter and spur gears show similar features except the addendum height of the pinion cutter is usually being 1.25×m instead of 1×m. The geometry of pinion cutter is illustrated in Figure 1.

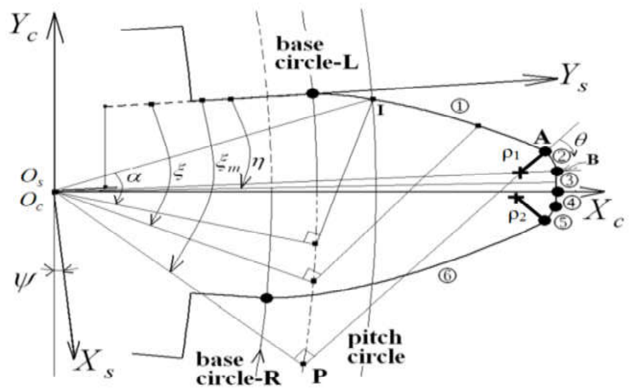


Figure 1. The geometry of pinion cutter

In illustrated figure; regions of 1-6 generate an involute section of the internal spur gear, 2-5 generates a trochoid section and 3-4 generates dedendum circle of spur gear. $S_s(X_s,Y_s)$ is reference coordinate system and $S_c(X_c,Y_c)$ is cutter coordinate system. In this study using of homogeneous coordinate system is seen appropriate. Where ψ is rotating angle, α is pressure angle, ξ is the involute angle and design parameter of region 1-6 changes $0 \le \xi \le \xi_m$, ξ_m is the maximum involute angle. The mathematical equation of region 1-6 is presented below;

$$R_c^1 = \begin{bmatrix} r_b \cos(\xi - \psi) + r_b\, \xi \sin(\xi - \psi) \\ -r_b \sin(\xi - \psi) + r_b\, \xi \cos(\xi - \psi) \\ 1 \end{bmatrix} \tag{1}$$

$$R_c^6 = \begin{bmatrix} r_b \cos(\xi - \psi) + r_b\, \xi \sin(\xi - \psi) \\ r_b \sin(\xi - \psi) - r_b\, \xi \cos(\xi - \psi) \\ 1 \end{bmatrix} \tag{2}$$

$$\psi = \frac{\pi}{2z_c} + tan(\alpha) - (\alpha) \tag{3}$$

Where r_b is the radius of the base circle and z_c is teeth number of pinion cutter.
The mathematical equation of region 2-5 is presented below;

$$R_c^2 = \begin{bmatrix} (r_b \cos(\xi_m - \psi) + r_b\, \xi_m \sin(\xi_m - \psi) \\ -\rho_1 \sin(\xi_m - \psi) + \rho_1 \cos(\theta_1 + \xi_m - \psi)) \\ (-r_b \sin(\xi_m - \psi) + r_b \xi_m \sin(\xi_m - \psi) \\ -\rho_1 \sin(\xi_m - \psi) + \rho_1 \cos(\theta_1 + \xi_m - \psi)) \\ 1 \end{bmatrix} \tag{4}$$

$$R_c^5 = \begin{bmatrix} (r_b \cos(\xi_m - \psi) + r_b \xi_m \sin(\xi_m - \psi) \\ -\rho_2 \sin(\xi_m - \psi) + \rho_2 \cos(\theta_2 + \xi_m - \psi)) \\ (r_b \sin(\xi_m - \psi) - r_b \xi_m \sin(\xi_m - \psi) \\ \rho_2 \sin(\xi_m - \psi) - \rho_2 \cos(\theta_2 + \xi_m - \psi)) \\ 1 \end{bmatrix} \tag{5}$$

Where Θ is design parameter of region 2-5 and changes $0 \le \Theta_{1,2} \le (\pi/2)-\arctan(\xi_m-(\rho_{1,2}/r_b))$. The mathematical equation of region 3-4 is presented;

$$R_c^3 = \begin{bmatrix} r_a \cos(\eta - \psi) \\ r_a \sin(\eta - \psi) \\ 1 \end{bmatrix} \tag{6}$$

$$R_c^4 = \begin{bmatrix} r_a \cos(\eta - \psi) \\ -r_a \sin(\eta - \psi) \\ 1 \end{bmatrix} \tag{7}$$

$$r_a = \sqrt{r_b + (r_b \xi_m - \rho)^2} + \rho \tag{8}$$

Where η is designed parameter of region 3-4 and changes $\xi_m+\delta-(\pi/2) \le \eta \le (\pi/2z_c)+\tan(\alpha)-(\alpha)$.

$$\delta = \frac{\pi}{2} - \arctan\left(\xi_m - \left(\rho_{1,2} - r_b\right)\right) \tag{9}$$

Normal vector of cutter regions are calculated by using differential geometry with the equation below;

$$n_c^i = \frac{\frac{dR_c^i}{dv_j} x k_c}{\left|\frac{dR_c^i}{dv_j} x k_c\right|} \tag{10}$$

Design Of Internal Spur Gear: In stage of manufacturing the cutter and internal spur gear blank are moving simultaneously. The kinematic relation between blank and cutter is showed in Figure 2.

The mathematical equations of the internal spur gear are obtained from the appropriate transformation equation. Coordinate transformation matrix is given with the equation below;

$$M_{gc} = \begin{bmatrix} \cos(\phi_g - \phi_c) & -\sin(\phi_g - \phi_c) & (r_{0g} - r_{0c})\cos(\phi_g) \\ \sin(\phi_g - \phi_c) & \cos(\phi_g - \phi_c) & (r_{0g} - r_{0c})\sin(\phi_g) \\ 0 & 0 & 1 \end{bmatrix} \tag{11}$$

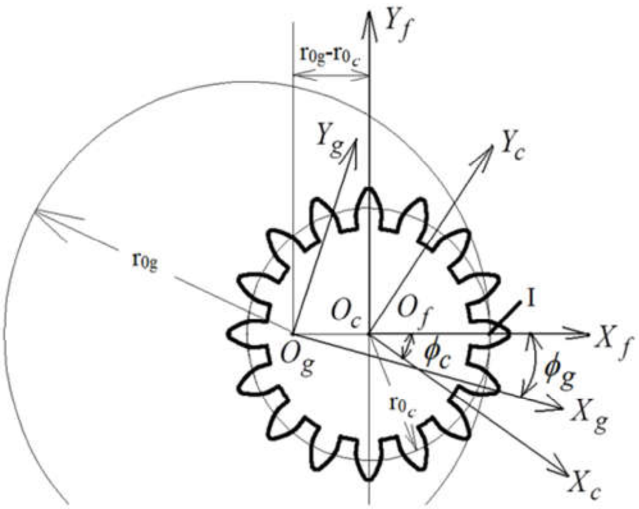


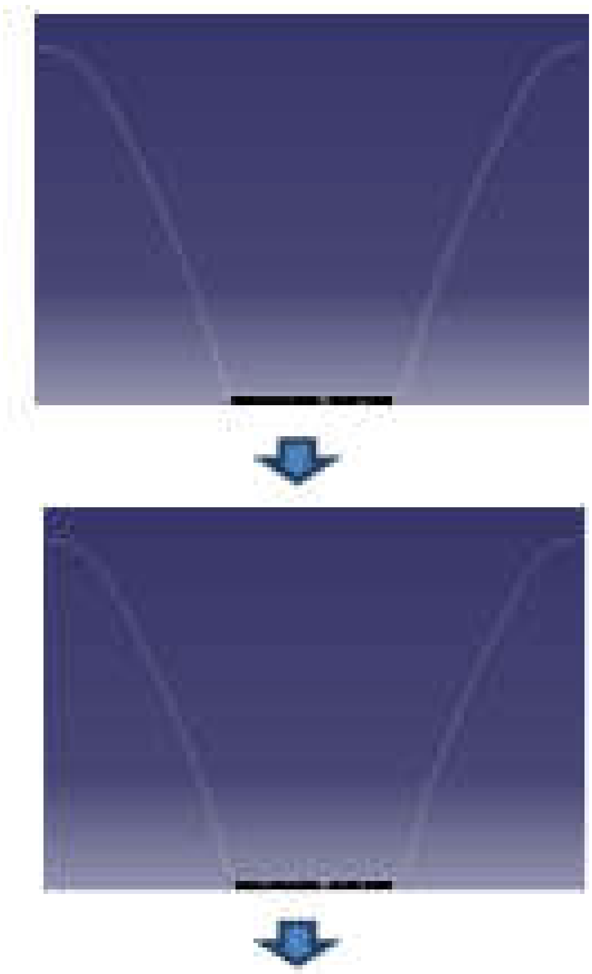
Figure 2. Kinematic relation between blank and cutter

Where $Ø_g$ and $Ø_c$ are rotating angle, r_{oc} is the radius of the pitch circle of the cutter and r_{og} is the radius of the pitch circle of the internal gear. The relation between rotating angles is presented with the equation below;

$$\phi_g = \left(\frac{r_{0c}}{r_{0g}}\right)\phi_c \tag{12}$$

The mathematical equation of internal spur gear is obtained from the equation below;

$$R_g^i = [M_{gc}]R_c^i \tag{13}$$



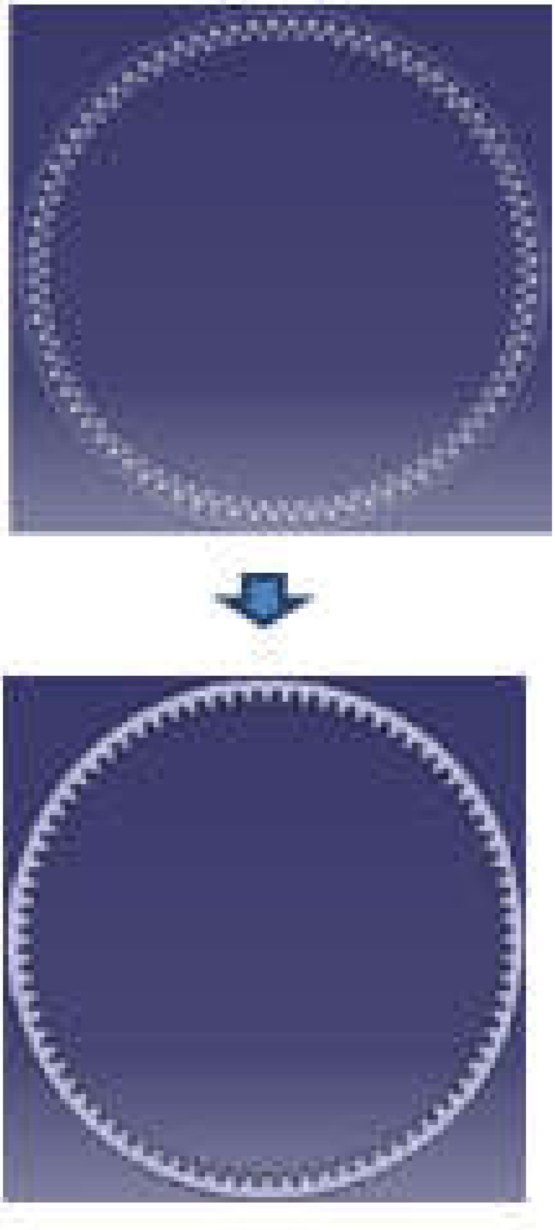


Figure 4. Design steps in CAD program

Table 1. Gear Properties

Proporties	*Internal*
Module (m)	3.18 mm
Number of teeth (z)	48
Pressure Angle ($\alpha_{c,d}$)	20º
Addendum coefficient (h_a)	0.8
Dedendum coefficient (h_f)	1.25
Tip radius coefficient of pinion cutter for	0.2-0.2
drive and coast side (ρ_{d-} ρ_c)	0.5-0.19
Profile shifting (x)	0
Rim thickness (mm)	1.5xm-5xm
Facewidth (b)	10 mm

The common normal vector of cutter and blank passes on instantaneous rotating center (I) according to gearing theory.
The mathematical expression of this phenomenon is presented with the equation below;

$$\frac{X_c - x_c^i}{n_{cx}^i} = \frac{Y_c - y_c^i}{n_{cy}^i} \qquad for\ i = 1 - 6 \tag{14}$$

Where $X_c = r_{0c}\cos(Ø_c)$ and $Y_c = r_{0c}\sin(Ø_c)$ are coordinates of I. x_c^i and y_c^i is the coordinates of cutter regions. n^i_{cx} and n^i_{cy} are the components of unit normal vector n_c^i. Internal spur gear points are obtained by solving (13) and (14) simultaneously. Design phases of internal spur gear are illustrated in CATIA and showed in Figure 4.

RESULTS AND DISCUSSION

Finite Element Analyses and Case Study: To obtain effect of asymmetric tip radius on bending stress of internal gear, finite element analyses were used for a selected thin rim thickness. Gear properties used in analyses is illustrated in Table 1. Asymmetric tip radius value is determined from previous study (Yilmaz et al., 2017). For analyses, three teeth internal gear finite element model was used. 3000 N mesh force is loaded from HPSTC. Load and boundary situation are presented in Figure 5. To allow rim deformation rim area are not fixed only lateral faces of model are fixed. In Figure 6 and 7 the bending stress value are illustrated for symmetric tip radius. In Figure 8 and 9 the bending stress values for asymmetric tip radius are illustrated.

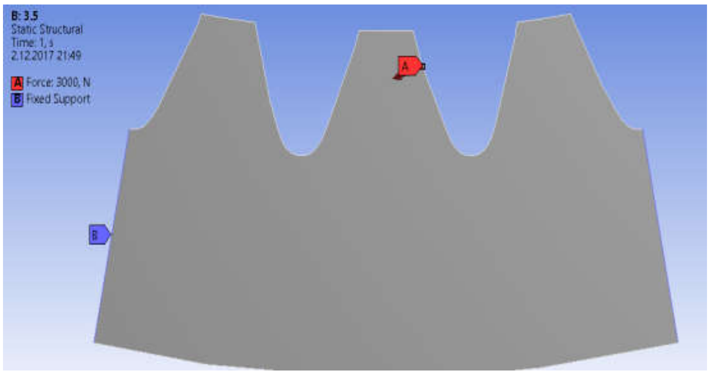


Figure 5. Load and boundary situation

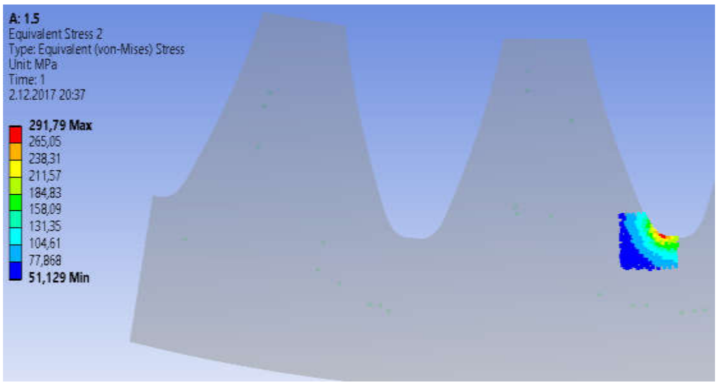


Figure 6. Bending stress result for 0.2-0.2xm tip radius and 1.5xm rim thickness

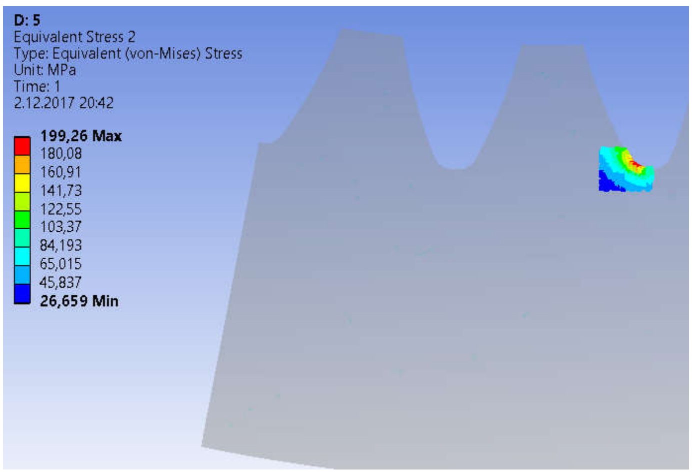


Figure 7. Bending stress result for 0.2-0.2xm tip radius and 5xm rim thickness

According to results the internal spur gear with asymmetric trochoid has lower stress for same rim thickness. For symmetric tip radius, when rim thickness is increased from 1.5 to 5 the stress value decreases for about 31.7%. For asymmetric in same manner, the value decreases approximately 32.8%. For 1.5xm rim thickness, asymmetric concept has 15% lower bending stress when comparing symmetric concept. For 5xm rim thickness, Asymmetric concept is better since it reduces bending stress for about 16.2% when comparing symmetric one.

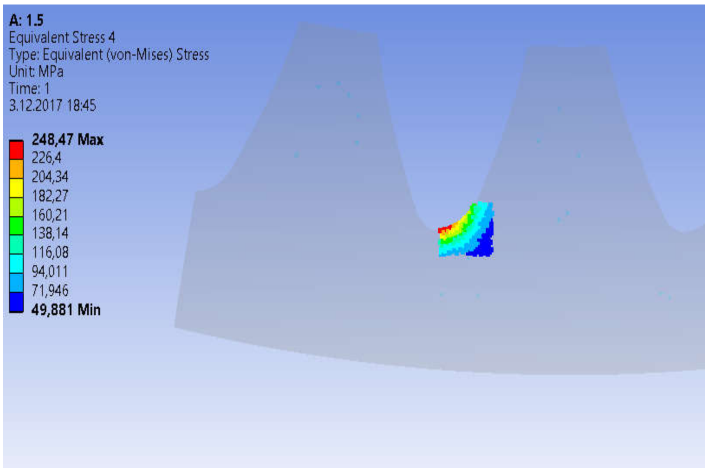


Figure 8. Bending stress result for 0.5-0.19xm tip radius and 1.5xm rim thickness

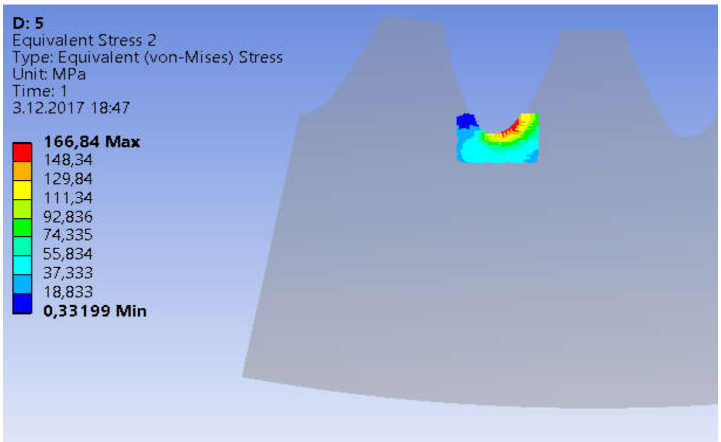


Figure 9. Bending stress result for 0.5-0.19xm tip radius and 5xm rim thickness

Conclusion

In this study, the effects of using symmetric and asymmetric cutter tip radius on bending stress of internal gear with different rim thickness. First the mathematical expression of cutter was set. Then using gearing theory and gear manufacturing kinematics, the coordinates of points of internal gear tooth was obtained. 3D design of gear is realized in CATIA. In ANSYS, finite element analyses were conducted for different rim thickness and tip radius. According to results increasing tip radius decreases root stress of gear. On the other hand, root stress decreases with increasing rim thickness. As a significant output; rim thickness is more effective on stress than tip radius.

REFERENCES

Buckingham, E. 1949. Analytical Mechanics of Gears, New York: Dover Publications, pp. 566.

Buckingham, E., Buckingham, E. 1999. Manual of Gear Design: Spur and Internal Gears, Industrial Press, New York, pp. 153.

Colbourne, J. R. 1987. The Geometry of Involute Gears, New Jersey: Springer-Verlag, pp. 526.

Colbourne, J. R. 1990. The Geometric Design of Internal Gear Pairs, *Gear Technology*, May/June: 28-37.

Fetvacı, C. 2016. Pinyon-Tipi Takımla Dişli İmalat Simülasyonunda Evolvent Parametresinin Efektif Sınırının Tayini, *Gazi Üniversitesi Mühendislik Mimarlık Fakültesi Dergisi*, volume 31, issue 2, pp. 449-455.

Jelaska, D. 2012. Gears and Gear Drives, UK: Wiley, pp. 472.

Karpat, F., Engin, B., Dogan, O., Yuce, C., Yılmaz, T. G. 2014. Effect of Rim Thickness on Tooth Root Stress and Mesh Stiffness of Internal Gears, *ASME 2014 International Mechanical Engineering Congress and Exposition,* 13-19 November 2014, Montreal, Canada.

Karpat, F., Ekwaro, S., Yılmaz, T. G., Dogan, O., Yuce, C. 2015. Design and Analysis of Internal Gears With Different Rim Thickness and Shapes, *ASME 2015 International Mechanical Engineering Congress and Exposition,* 14-20 November 2015, Houston, ABD.

Kawalec, A., Wiktor, J. 2004. Tooth-root stress calculation of internal spur gears, *Proceedings of the Institution of Mechanical Engineers, Part B: Journal of Engineering Manufacture*, volume 218, issue 9, pp. 1153-1166.

Litvin, F. L., Hsiao, C. L., Wang, J. C., Zhou, X. 1994. Computerized Simulation of Generation of Internal Involute Gears and Their Assembly, *Journal of Mechanical Design*, vol: 116, issue 3, pp. 683-689.

Litvin, F. L., Hsiao, C. L., Wang, J. C., Zhou, X. 1994. Computerized Simulation of Generation of Internal Involute Gears and Their Assembly, *Journal of Mechanical Design*, volume 116, issue 3, pp. 683-689.

Muni, D. V., Muthuveerappan, G. 2009. A Comprehensive Study on the Asymmetric Internal Spur Gear Drives through Direct and Conventional Gear Design. *Mechanics Based Design of Structures and Machines,* volume 37, issue 4, pp.431-461.

Oda, S., Miyachika, K., Araki, K. 1984. Effects of Rim Thickness on Root Stress and Bending Fatigue Strength of Internal Gear Tooth, *Bulletin of JSME*, volume 27, issue 230, pp. 1759-1764.

Oda, S., Miyachika, K. 1986. Practical Formula for True Root Stress of Internal Spur Gear Tooth, *Bulletin of JSME*, volume 29, issue 252, pp. 1934-1939.

Radzevich, S. P., Darle, W. D. 1994. Handbook of Practical Gear Design, CRC press, USA, pp. 847.

Sanchez, M. B., Pleguezuelos, M., Pedrero, J. 2016. Calculation of tooth bending strength and surface durability of internal spur gear drives, *Mechanism and Machine Theory,* volume 95, pp. 102-113.

Yang, S. C. 2007. Study on an Internal Gear with Asymmetric Involute Teeth, *Mechanism and Machine Theory*, volume 42, issue 8, pp. 977-994.

Yılmaz, T. G., Dogan, O., Yuce, C., Karpat, F. 2017. Improvement Of Loading Capacity Of Internal Spur Gear With Using Asymmetric Trochoid Profile. ASME 2017 International Mechanical Engineering Congress and Exposition, 3-9 November 2017, Tampa, ABD.

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