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ON GENERALIZED RELATIONS

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ABSTRACT

The article summarizes the relationships introduced by Purdea [1] and Goghen [2]. Goghen gives a summary of L - the relationships examined by Salius and the fuzzy relations of Zade and Purdea - of all known other types of relations. The terminology of Wagner [5] and Bourbaki [6] is used.

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INTRODUCTION

Let F be a function with a definition domain, the set T, and the functional values are the set F(t) ⊂ x for ∀ t ∈ T.

We denote by $\underset{t\in T}{\Pi F}(t)$the cartesian product of the family the sets F (t) indexed by the elements of the set T, B - a set of sets, K - a set with or without relationships and operations. Let R be a function with a definition domain $\underset{T\in Bt\in T}{\cup\Pi F}(t)$and functional values of K.

Definition 1. Generalized relations R of type (B, K) between sets of elements $F(t), t\in T, T\in B$is the triad $\underset{T\in Bt\in T}{\cup\Pi F}\ F(t), K, R$

If F(t) = Ø for any t ∈ T, T ∈ B, or K = Ø, then R = Ø.

Private cases of such a generalized relation are:

1) If K is a non-empty set and T ∈ B rearranged sets, then the generalized relation coincides with the generalized relation of Purdea [1].
2) If K is the single interval [0,1] with the known addition, subtraction, multiplication, ordinance, and if $B=T, T=\{t_1,t_2\}$ the generalized relation coincides with the fuzzy relation defined by Zade in [2].
3) If instead of the single interval K = [0,1] set K = L - a partially ordered set, we obtain the relations examined by Gogens in [2].
4) L - the relations defined by Salij in [3], are obtained at K = L, L - lattice.

If F (t) = F for ∀ t ∈ T, T ∈ B the generalized relation is called homogeneous.
Let B1 be a family of sets $T_1\subset T, T\in B$ and R1 is a restriction of R.

Thetriad$pr_{T_1\in B}(B,K)=\underset{T\in Bt_1\in T_1\subset T}{\cup\Pi F}F(t_1), K, R$ is called projection.

If T1 = T for $\forall T \in B$, the projection is called non-proprietary, but if $T_1 T_1 \neq T \; and \; T \in B$ for some - own.
Let B = {T} and T_1= {t} ⊂ T, then $Pr_{T_1}(B, K)$ coincide with the restriction of the function R on the base set F (t).
Let's $\sigma = \{\sigma_t / T \in B\}$ be a family of bigections $\sigma_t : T \longrightarrow T$, for at least one T ∈ B is σ_T not the same.
The generalized relation R^{σ} we have: $((x_{t\in T, T\in B, K)} \in R \Leftrightarrow ((x_{\sigma t\in T, T\in B, K)} \in R^{\sigma} \forall t \in T$

σ is called σ-inverse relation of R. If K is a non-empty set and P reordered sets, this definition coincides with the same definition of Purdea [1] from which is obtained as a private case (i, j), the transposition of Penzow [8].

Let the binary operations V (defined on subsets) and * be defined on the set K, such that:

1. The summary of the Birkhoff law [7] is in force for V: $\underset{i}{V} \underset{j \in \Phi_i}{V} a_j = \underset{j \in \Phi}{V} a_j$

$\Phi = \underset{i}{U} \Phi_i, \Phi_i$, - a plurality of indices

From this law follows Idempotent, Commutative and Associate for V - Birkhof [7]
2. * is associative and has 0 and 1;
3. the two complete distributive laws link V and *
$a * V_i b_i = V_i(a * b_i), V a_i * b = V_i(a_i * b)$

equivalent to equality [2], $\underset{i \in \Phi}{V} a_i * \underset{j \in \Psi}{V b_j} = \underset{(i,j) \in (\Phi, \Psi)}{V(a_j * b_j)}$ p. 152, proposal 2);
4. 0Vk = k and 1Vk = 1

These conditions are satisfied, for example, for K - a complete structured semigroup (Goghen, [2]).
Let me
$R_i = \left(\underset{T_i \in B_i}{\cup} \underset{t_i \in T}{\prod} F(t_i), K, R_i\right)$ and $R_j = \left(\underset{T_j \in B_j}{\cup} \underset{t_j \in T}{\prod} F(t_j), K, R_j\right) T_i \cap T_j = \phi$

are two generalized relationships. We denote: $W^{T_k}_{R_i \circ R_j}, V^{T_k}_{R_i \circ R_j}, X^{T_k}_{R_i \circ R_j}, k = i, j$

three non-intermittent sets for which $W^{T_k}_{Ri \circ Rj} U V^{T_k}_{Ri \circ Rj} U X^{T_k}_{Ri \circ Rj} = T_k, k = i, j$

G - a family of surections

$$g_{Ri \circ Rj} : T \longrightarrow T_{Ri \circ Rj} \subset T = T_i \cup T_j \in B_{R_i \circ R_j} = B_{R_i} \cup B_{R_j},$$

$$g\left(W^{T_k}_{R_i \circ R_j}\right) \cap g\left(X^{T_k}_{R_i \circ R_j}\right) = \phi, g\left(V^{T_k}_{R_i \circ R_j}\right) \cap . g\left(W^{T_k}_{R_i \circ R_j}\right) = \phi, g\left(W^{T_k}_{R_i \circ R_j}\right) \cap g\left(X^{T_k}_{R_i \circ R_j}\right) = \phi, k = i, j, g\left(W^{T_i}_{R_i \circ R_j}\right) \cap g\left(W^{T_j}_{R_i \circ R_j}\right) = \phi, g\left(V^{T_i}_{R_i \circ R_j}\right) = g\left(V^{T_j}_{R_i \circ R_j}\right), g\left(X^{T_i}_{R_i \circ R_j}\right) = g(X^{T_k}_{R_i \circ R_j});$$

H - the subfamily of G formed by the restrictions $h_{R_i \circ R_j}$ of g on;

$$W^{T_i}_{R_i \circ R_j} \cup W^{T_j}_{R_i \circ R_j} \cup V^{T_i}_{R_i \circ R_j} \cup V^{T_j}_{R_i \circ R_j}, P_{R_i \circ R_j} = h_{R_i \circ R_j}(T_i \cup T_j); \; C^T_q = \frac{\Pi F(s)}{seg^{-1}_{R_i \circ R_j}(q)}, q \in T_{R_i \circ R_j}$$

It is supposed $q \in q^{-1}_{R_i \circ R_j}(g)$, to not reduce the community.

Definition 2. The product $R_{i \circ} R_j$ of the type (G, H, B) of the relations R1 and Rj is determined by the equation:

$$R_{j \circ} R_j = \left\{\left[(c_p)_{p \in P_{R_i \circ R_j}} {}'k, R_i \circ R_j\right] / k = \underset{t}{V}\left(k_{R_i} * k_{R_i}), (x_t)_{t \in g^{-1}(T_{R_i \circ R_j} / P_{R_i \circ R_j})} / A(R_i \circ R_j)\right)\right\},$$

Where/ 1 /

$$A(R_i, R_j) \equiv [k_{R_s} = R_s(x_{t_s}) t_s \in T_s, s = i, j; \left(g_{R_i \circ R_j}(t_k) = g_{R_i \circ R_j}(t_l) \Longrightarrow x_{t_k} = x_{t_l}\right); \left(g_{R_i \circ R_j}(t) = p \in P_{R_i \circ R_j} \Longrightarrow x_t = c_p \in C^T_p, T \in B_{R_i \circ R_j}\right)]$$

(We accept:) $T_{R_i \circ R_j} = P_{R_i \circ R_j} \Longrightarrow k = k_{R_i} * k_{R_j}$

If for any p we have $C_p = \phi$, then $R_{i \circ} R_j = \phi$
In the case of B = {T} and T$\{t_1, t_2\}$, the product $R_{i \circ} R_j$ coincides with the work of Goghen [2], p. 161.
Let T ∈ B multitudes be rearranged, and K is a non-empty set: $k_1 * k_2 = \begin{cases} k, \; if \; k_1 = k_2 = k \\ y, \; if \; k_1 \neq k_2 \end{cases},$

then definition 2 coincides with definition 1 given by Purdea in [1].

The case $K = \{k, y\}, k = 1, y = 0, X^{T_i}_{R_{i_\circ} R_j}$ и $\underset{XRi \circ Rj}{Tj}$ - isomorphic coincides with definition 8 given by Nemety [9].

A particular case from the Purdea definition is the definition of (r, s) - a product of two inhomogeneous n - relationships introduced in [10] by Topencharov, and for the homogeneous n relations introduced in [8] by Penzov.

Let be given $Ri = (\underset{Ti \in B}{\cup} \underset{iti \in Ti}{\Pi} F(ti), K, Ri), Ti \cap Tj \neq \emptyset$

$i, j = 1,2,3, i \neq j$- three generalized relations. We continue $g_{R_{1_\circ} R_2}$ and $g_{R_{2_\circ} R_3}$ on

$$T_1 \cup T_2 \cup T_3 = T \in B = B_1 \cup B_2 \cup B_3 : \ g_{R_{1_\circ} R_2} : T \Longrightarrow T_{R_{1_\circ} R_2}, T_{R_{1_\circ} R_2} \subset T, g_{R_{1_\circ} R_2}(t_3) = t_3, t_3 \in T_3, g_{R_{2_\circ} R_3} : T \Longrightarrow T_{R_{2_\circ} R_3}, T_{R_{2_\circ} R_3} \subset T, g_{R_{2_\circ} R_3}(t_1) = t_1, t_1 \in T_1$$

and apply to the products $(R_{1_\circ} R_2)_\circ R_3$ $and R_{1_\circ}$ $(R_{2_\circ} R_3)$

$g_{R_{1_\circ} (R_{2_\circ} R_3)} = g_{R_{1_\circ} R_2}$ and $g_{(R_{1_\circ} R_2)_\circ R_3} = g_{R_{2_\circ} R_3}$

We mean:

$g_{(R_{1_\circ} R_2)_\circ R_3}\left(T_{R_{1_\circ} R_2}\right) = T_{(R_{1_\circ} R_2)_\circ R_3}$,

$g_{R_{1_\circ} (R_{2_\circ} R_3)}\left(T_{R_{2_\circ} R_3}\right) = T_{R_{1_\circ} (R_{2_\circ} R_3)}$

We assume the fulfillment of the important conditions:

/ 2 / $g_{(R_{1_\circ} R_2)_\circ R_3 \circ} g_{R_{1_\circ} R_2} = g_{R_{1_\circ} (R_{2_\circ} R_3) \circ} g_{R_{2_\circ} R_3}$;

/ 3 / $X^{T_2}_{R_{1_\circ} R_2} \cap X^{T_2}_{R_{2_\circ} R_3} = \phi$

Then the following applies

Theorem: $(R_{1_\circ} R_2)_\circ R_3 = R_{1_\circ}$ $(R_{2_\circ} R_3)$

Proof:

$$k_{(R_1 \circ R_2) \circ R_3} = (R_1 \circ R_2) \circ R_3 \left((c_p)_{p \in P_{(R_1 \circ R_2) \circ R_3}} \right) = \underset{t}{V} (k_{R_1 \circ R_2} * k_{R_3}) / (x_t)_{t \in g^{-1}_{(R_1 \circ R_2) \circ R_3}} \left(T_{(R_1 \circ R_2) \circ R_3} \backslash P_{(R_1 \circ R_2) \circ R_3} \right) / A\left((R_1 \circ R_2) \circ R_3\right)$$

$$= \underset{t}{V} \left(\underset{t_1}{V} (k_{R_1} * k_{R_3}) * k_{R_3} \right) / \left((x_{t_1})_{t_1 \in g^{-1}_{(R_1 \circ R_2) \circ R_3}\left(T_{(R_1 \circ R_2) \circ R_3} \backslash P_{(R_1 \circ R_2) \circ R_3}\right)} / A(R_1 \circ R_2) \right),$$

$$\left((x_{t_1})_{t_1 \in g^{-1}_{(R_1 \circ R_2) \circ R_3}\left(T_{(R_1 \circ R_2) \circ R_3} \backslash P_{(R_1 \circ R_2) \circ R_3}\right)} / A(R_1 \circ R_2) \right)$$

$$= \underset{t}{V} [k_{R_1} * (k_{R_2} * k_{R_3})]$$

$$/ \left((x_t)_{t \in g^{-1}_{(R_2 \circ R_3)}} \circ g^{-1}_{R_1 \circ (R_2 \circ R_3)} \left(T_{R_1 \circ (R_2 \circ R_3)} \backslash P_{R_1 (R_2 \circ R_3)} \right) / A(R_1 \circ R_2) \right), \left(k_{R_3} = R_3 \left(x_{t_{R_3}} \right)_{t_{R_3} \in T_3}, g_{(R_1 \circ R_2) \circ R_3}(t_k) = g_{(R_1 \circ R_2) \circ R_3}(t_l) \right) \Rightarrow x_{t_k} = x_{t_l}, \left(g_{(R_1 \circ R_2) \circ R_3}(t) = p \in P_{(R_1 \circ R_2) \circ R_3} \Rightarrow x_t = c_p \in C_P^T \right)$$

$$= \underset{t}{V} [k_{R_1} * (k_{R_2} * k_{R_3})] / \left((x_t)_{t \in g^{-1}_{(R_2 \circ R_3)}} \circ g^{-1}_{R_1 \circ (R_2 \circ R_3)} \left(T_{R_1 \circ (R_2 \circ R_3)} \backslash P_{R_1 \circ (R_2 \circ R_3)} \right) / A(R_2 \circ R_3) \right),$$

$$k_{R_1} = R_1 \left(x_{t_{R_1}} \right)_{t_{R_1}} \in T_1, [g_{R_1 \circ R_2}(t_k) = g_{R_1 \circ R_2}(t_l) \Rightarrow x_{t_k} = x_{t_l}], g_{R_1 \circ R_2}(t) = \in P_{R_1 \circ R_2} \Rightarrow x_t = c_p \in C_P^T$$

$$= \underset{t}{V} \left\{ k_{R_1} * \left[\underset{t_2}{V} \left((k_{R_2} * k_{R_3}) j \right) \right] \right\} / \left((x_{t_2})_{t_2 \in g^{-1}_{(R_2 \circ R_3) \circ R_3}\left(T_{R_2 \circ R_3} \backslash P_{R_2 \circ R_3}\right)} \right)$$

$$/ A(R_2 \circ R_3), (x_t)_{t \in g^{-1}_{R_1 (R_2 \circ R_3)}} \left(T_{R_1 (R_2 \circ R_3)} \backslash P_{R_1 (R_2 \circ R_3)} \right) / A\left((R_2 \circ R_3)\right), k_{R_1} = R_1 (x_{t_1})_{t_1 \in T_1}, g_{R_1 \circ (R_2 \circ R_3)}(t_k) =, g_{R_1 \circ (R_2 \circ R_3)}(t_l) \Rightarrow x_{t_k} = x_{t_l}, g_{R_1 \circ (R_2 \circ R_3)}(t) = p \in P_{R_1 \circ (R_2 \circ R_3)} \Rightarrow x_t = c_p \in C_P^T$$

$$= \underset{t}{V} (k_{R_1} * k_{R_2 \circ R_3}) / \left((x_t)_{t \in g^{-1}_{R_1 \circ (R_2 \circ R_3)}} \left(T_{R_1 \circ (R_2 \circ R_3)} \backslash P_{R_1 \circ (R_2 \circ R_3)} \right) / A(R_1, R_2 \circ R_3) \right) = k_{R_1 \circ (R_2 \circ R_3)},$$

which we had to prove.

In [6, 7, 8] we use equations 1, 2, 3, the associativity of K regarding * and the summary distribution laws concerning V.

The theorem we examined is also true for intersectingT_1, T_2, T_3. Instead of $T_1, T_2$and T_3, the sets $T_1 = (T_1, 1), T_2 = (T_2, 2), T_3 = (T_3, 3)$ which do not intersect and are equal to respectively T_1, T_2 and T_3.

For the generalized relations $R_{1\circ} R_2$ and R_2, R_3, the functions $g_{R_1 \circ R_2}$ and $g_{R_2 \circ R_3}$ are for $\forall T$ are bijections and

$$g_{R_1 \circ R_2}\left(W^{T_1}_{R_1 \circ R_2}\right) = W^{T_1}_{R_1 \circ R_2}, g_{R_1 \circ R_2}\left(V^{T_1}_{R_1 \circ R_2}\right) = g_{R_2 \circ R_3}\left(V^{T_3}_{R_2 \circ R_3}\right) = V^{T_2}_{R_2 \circ R_3}, g_{R_2 \circ R_3}\left(W^{T_2}_{R_2 \circ R_3}\right) = g_{R_2 \circ R_3}\left(X^{T_3}_{R_2 \circ R_3}\right)$$
$$= W^{T_2}_{R_2 \circ R_3}, g_{R_1 \circ R_2}\left(X^{T_1}_{R_1 \circ R_2}\right) = W^{T_2}_{R_2 \circ R_3}, g_{R_1 \circ R_2}(W^{T_2}_{R_1 \circ R_2}) \sim W^{T_2}_{R_2 \circ R_3}$$

$k_{R_2} = R_2(x_t)t \in T_2 = \{1,$

If $x_l = x_m = x_n = x_p, g_{R_1 \circ R_2}(l) = g_{R_1 \circ R_2}(m), g_{R_2 \circ R_3}(n) = g_{R_2 \circ R_3}(p), x_{s_1} = x_{s_2} = x_{s_3}, g_{R_1 \circ R_2}(s_1) = g_{R_1 \circ R_2}(s_2) = g_{R_2 \circ R_3}(s_2) = g_{R_2 \circ R_3}(s_3),$

and 0 otherwise.

Under these conditions

$k_{R_1 \circ R_2} = k_{R_1} * k_{R_2} / \left((x_t)_{t \in g^{-1}_{R_1 \circ R_2}}\left(T_{R_1 \circ R_2} \backslash P_{(R_1 \circ R_2)}\right) / A(R_1, R_2)\right) = k_{R_1} / \left((x_t)t \in T_1\right), k_{R_1 \circ R_2} = k_{R_1}$

Similarly displayed $k_{R_2 \circ R_3} = k_{R_3}$. It follows:

Theorem R2. The relations satisfying the above conditions is a right unit for R1 and a left unit for R3.

From theorem 1 and theorem 2 follows:

Theorem 3. The aggregate of the generalized relations, for which $g_{R_1 \circ R_2}$and $g_{R_2 \circ R_3}$are biections, is a category.
The aggregate of the generalized relations, for which $\boldsymbol{g_{R_1 \circ R_2}}$and $\boldsymbol{g_{R_2 \circ R_3}}$are biections, is a category.
Data stored on the computer is called a database [11-12]. Typically, the data in the computer is represented in tables. Each table represents n-ary relationship.
To extract information and to modify the content of the tables, corresponding to a set of relationships, some of the basic operations on them are defined, namely: "Projection", "Compound", and "Select".
An operation "Compound" merges two tables into a larger table:
If, $R \subset (A_1 X \dots\dots\dots X A_m \times B_1 X \dots\dots\dots X B_n)$ and
$S \subset \left(A_1 X \dots\dots\dots X A_m \times C_1 X \dots\dots\dots C_p\right)$
this compound $\boldsymbol{R}$and $\boldsymbol{S}$ are:

$$\subset \left(A_1 X \dots\dots\dots X A_m \times B_1 X \dots\dots B_n \times C_1 X \dots\dots\dots X C_p\right)$$

e.g. the compound consists of elements of the type:
$\left(a_1, \dots\dots, a_m, b_1 \dots\dots., b_n, c_1, \dots\dots, C_p\right)$,
where:
$(a_1, \dots\dots, a_m, b_1 \dots\dots., b_n) \in R$
whereas
$(a_1, \dots\dots, a_m, b_1 \dots\dots., b_n) \in S$
The operation "Projection" forms a new table ($\boldsymbol{k}$ - ratio) from certain columns of the old table ($\boldsymbol{n}$ - ratio) if $\boldsymbol{k} < n$.
The operation "Select" chooses rows of the table that satisfy appropriate criteria.

Literature

1. Purdea, I. Relations generalisees. Rev. Roum, Math. Pureset Appl. 1969, 14, 4, 533-556.
2. Goghen, J. L-Fuzzy sets. Jour, Math. Anal. and Applic. 1967, vol. 18, 145-174
3. Салий, В.Н.Бинарные∠ - отношения. Изв. ВУЗ, Математика, 1965, 1, 133-145.
4. Zadeh, L. A. Fuzzy sets, Information and Control, 1965, vol. 8, 338-353.
5. Вагнер, В.В. Теорияотношений и алгебрачастичныхотображений, Теорияполугрупп иееприложения. Сборникстатей, выпуск 1, Изд. Саратовскогоуниверситета, Саратов, 1965.
6. Бурбаки, Н. Теориямножеств. М, Физматгиз, 1958.
7. Биркхоф, Г. Теорияструктур. М., ИЛ, 1952.
8. Пензов, Ю.Е. К арифметике∏ - отношений. Изв. ВУЗ Математика, 1961, 23, 4, 78-92.
9. Nemethi, C. Despreprodusulrelatilor. St. cerc. mat. 1972, 24, 9, 1421-1434.
10. Топенчаров, В.В. Еднабинернакомпозицияна∏ - арни отношения, Год. на ВТУЗ, Математика, 1973, т.IX, кн.4.Jordan Milev-TU Sofia
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