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# FILIPINO STUDENTS' USE OF METACOGNITIVE SKILLS IN MATHEMATICAL PROBLEM SOLVING: AN EMERGENT MODEL

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**ABSTRACT**

This research draws a theoretical framework that underpins the mathematical problems solving heuristics among Filipino high school students through an evolved grounded theory. The central construct that emerged was the students' use of metacognitive skills in problem solving with five main processes that encompass an emerging substantive theory namely: understanding the problem through sense-making, organizing and constructing useful information from the problem; planning solution strategies by identifying, conjecturing and selecting strategies; executing the plan; checking the process and strategies undertaken; and reflecting and extending the problem. This model suggests ways of facilitating the development of Filipino students' problem solving heuristics.



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## INTRODUCTION

Problem solving has long been recognized as one of the hallmarks of mathematics. One of the greatest goals of mathematics education is to have students become good problem solvers (Billstein, Libeskind and Lott, 2000). Mathematics educators recognize the need to develop critical and analytic thinking through problem solving (Limjap and Candelaria, 2002). The 1989 Curriculum and Evaluation Standards for School Mathematics (NCTM, 1989) reflected a clear vision of the focal place of problem solving in Mathematics. Also, the 2000 Principles and Standards for School Mathematics reiterate the central place of problem solving at all levels of mathematics as follows:

*By learning problem solving in mathematics, students should acquire the ways of thinking, habits of persistence and curiosity, and confidence in unfamiliar situations that will serve them well outside the mathematics classroom. In everyday life and in the workplace, being a good problem solver can lead to great advantages (NCTM, 2000, p. 52).*

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According to Krulik and Rudnick (1996), problem solving is the means by which an individual uses previously acquired knowledge, skills, and understanding to satisfy the demands of an unfamiliar situation. It begins with the initial confrontation and ends when an answer has been obtained and checked against the condition of the problems. They advocated the teaching of problem solving in the classroom. The process of solving problems has been presented as a series of steps, referred to as a *heuristic plan*, or simply, *heuristics*. A heuristics plan provides a "road map" or a blueprint that directs one's path toward a solution and resolution of a problem situation. However, direct instructions do not specify this plan completely. In fact, students are not expected to give solutions automatically in problem solving. The task of solving the problem should be new to the individual, although processes or knowledge already available can be called upon for solution (Resnick and Glaser, 1976). A problem might be a genuine problem for one individual but might not be for another (Schunk, 2000). Although heuristics of mathematical problem solving has been described in international literature, nothing is written as of the time of this research on Filipino high school students' problem solving heuristics. Besides, these heuristics presented in literature is so general that their effectiveness is questionable when applied to the infinite variety of mathematical problems (Kalomitsines, 1983). Thus the substantive area of this study was chosen to address the interest of the researchers, purpose and initial inquiry with the intention to formulate a model of the problem solving heuristics specifically of Filipino senior high school students in Central Mindanao University. Using Grounded Theory (GT), this substantive theory is generated from the responses of the students to problems which their teachers believe are non routine and cannot be automatically solved by their students. According to Glaser, (1998), a GT proposal needs only to supply information on the area of interest, the data source and a statement of method to the effect that the researcher begins to collect, code, and analyze the data and let the theory emerge. This study includes the whole methodological package of GT for the readers to understand and appreciate it. This paper presents the portion of the research investigation that describes in detail the emergence of the integrative construct of core categories from the initial framework of the model.

## MATERIALS AND METHODS

A detailed research methodology section is unnecessary in GT because as a general method of inquiry, GT can be used for any substantive area and can work with all types of data and is already well documented (Glaser 1978, 1992, 1998).The constant comparison method by Strauss and Corbin (1967) and Pandit's (1996) creation of a theory methodology which draws concepts from Strauss and Corbin's (1998) theory building approach and Yin's (1989) case study approach guided this study. The procedures of grounded theory building are discussed followed by explanations of the procedures that were employed in this study. The methodology of Pandit's (1996) grounded theory consists of five phases namely: (1) research design, (2) data collection, (3) data ordering, (4) data analysis and (5) literature comparison. He stressed that these phases are not strictly sequential. Moreover, he also identified nine steps or procedures inherent in the said phases namely: (1) Review of Technical Literature, (2) Selecting Cases, (3) Development of Rigorous Data Collection Protocol, (4) Entering the field, (5) Data Ordering, (6) Analyzing Data Relating to the First Case, (7) Theoretical Sampling, (8) Reaching Closure and (9) Comparing Emergent Theory with Extant Literature. Details of the procedure will be discussed as the results are presented.The participants of this study were selected from the 100 senior students of a total population of about 500 students from Central Mindanao University High School. Using theoretical sampling technique, students were chosen based on their willingness to participate in the investigation. This technique was used to ensure manifestation of the theoretical construct of interest under investigation. For the initial model, the first case was chosen based on specified attributes.

**The Use of Grounded Theory in Data Analysis**

Grounded theory research, often referred to as the constant comparative method (Glaser and Strauss, 1967), is a qualitative tradition built on compared concepts. Proponents of the constant comparative method suggest that similar data are grouped and conceptually labeled. Then these concepts are categorized. Categories are linked and organized by relationships, conditions and dimensions are developed, and finally a theory emerges (Glaser, 1978; Glaser and Strauss, 1967; Strauss and Corbin, 1990; Laguda, 2007).

There is wide discussion of this method, and yet Boeije (2002) claims that the process for carrying out the analysis has remained vague. While a lack of specificity allows for creativity in the art and science of grounded theory research (Strauss and Corbin, 1998), it can mystify the novice researchers (Mc Caslin and Scott, 2003), and test them to trust themselves in the use of the grounded method, specifically their ability to generate codes and find relevance as much as possible without preconceived hypothesis (Laguda, 2007). Separately, Boeije (2002), McCaslin and Scott (2003), and Scott (2002) suggest additional rigor in data analysis to increase systematization and traceability. All three reports focus on comparative questions. Strauss and Corbin (1998) suggest that grounded theory analyst work to "uncover relationships among categories…by answering the questions of who, when, why, how and with what consequences…to relate to structure with process" (p.127), but do not specify how that is to be accomplished. The research of Laguda (2007) explicates a method for engaging those investigative questions effectively from relational linkages that bridge from analysis to interpretation and theory generation in grounded theory research. On the other hand, this research made use of those investigative questions which are applicable on the data. The primary researcher entered the field to conduct semi structured interview and to observe classes. The retrospective interview proceeds with only one defined question to be asked. The rest is an unstructured rapport-question-answer interaction. The initial question is "How do you solve mathematical problems?"

Then a probing question is asked in case the response is very technical as follows: "What are the processes you usually use when solving math problems?" Since rapport has to be built among the students and the interviewer, there were other questions asked such as "How do you select a strategy to solve a problems?" Theory may be generated from the data or existing theories may be elaborated and modified by the data. Both of these theory development processes were used in this study. An advantage to theory development is the researcher's reliance on three major types of data, which are namely: 1) semi-structured interviews, 2) direct observation and participants' problem solving output, and 3) extant literature over a period of time. Both expected and unexpected results were produced in the over-all data, requiring the researcher to search out and integrate concepts, eventually developing a theory that is grounded in data, observations and contemporary concepts. The word problems that the participants solved include numbers and operations, algebraic problem on sets, geometry problem involving solids, measurement problem involving rectangle and circle, probability problem, and logic problem. These problems were written in English and translated in Cebuano, the dialect of the students.

**Constant Comparative Method in Four Stages**

This research adapted the systematized constant comparison method presented by Glaser and Strauss (1967) in the analysis of the first case as well as the rest of the cases to discover the emergent themes and categories. The first stage of constant comparison began by comparing data with a group of incident in a single interview/sector to form categories or what is usually called open coding. Categories were defined, expanded and created as new information emerged. Categories were labeled according to the most appropriate codes.

From the categories, it was possible to formulate the core message from a single incident (Boeije, 2002; Laguda, 2007). The second stage of constant comparison involved analyzing a group of incidents in various interviews and comparing them with each other. Categories were continually defined, expanded and created as new information emerged. A code tree or inventory of characteristics of each category was created. Finally, categories were pooled and examined to see if they could be combined, integrated, or eliminated (Laguda, 2007). The third stage consisted of comparing sector one with sector two and so on and so forth. Categories were refined, short-listed, and developed through selective coding and sorting memos until they become saturated or “so well defined that there was no point in adding further exemplars to them” (Lauffer, 2002, p.101). From this, an emerging story line could emerge. The fourth stage involved analysis where explanatory and predictive patterns emerged and the writing begins. It is important to note that more than one pattern existed in numerous incidents. Despite the existence of more than one pattern, however, one overall pattern remains dominant. The dominant pattern in each incident was easily identified by the number of times characteristics of that pattern were present in a given incident. Laguda (2007) stressed that in this stage characteristics of the other non-dominant pattern occurred often, but not as often as those of the dominant pattern. The Cebuano speaking researcher analyzed the raw data in Cebuano and then translated the data in English. Then both researchers analyzed the English data according to the techniques and principles explained by Strauss and Corbin (1998). Moreover, consultations with some experts in the field were done to minimize if not eliminate bias in the coding of data.

## RESULTS

### Developing the Initial Framework

In this section, the initial framework derived from the analysis of the first case is presented. Prior to the presentation of the analysis, a description of the problem solving heuristics of the first case, an overview of how the analysis is done and the literatures in which the analysis are anchored are discussed and presented.

### Describing the Problem Solving Heuristics of the First Case

Case One’s problem solving heuristics is a cycle of analyze-solve-and-check phases with constant reflection in every step of the problem solving process. He usually engages in self-talk, self-questions, self-monitor, and self-evaluates every step he makes which indicates that he uses his metacognitive skills during the problem solving process. When interviewed, he expressed certain beliefs about problem solving in mathematics that influenced how he behaved when confronted with such tasks. He believes that he can solve the problem correctly when he can derive/recall formula(s) after careful analysis of the problem and do the necessary operations cautiously. These are the steps found in Polya’s (1945) problem solving framework in his famous book “How to Solve it”. According to Polya, the solvers have to understand the problem first and determine what is required in the problem. Then, they have to find out how the various items are connected, how the unknown and the given are linked, in order to make a plan. Next, they have to carry out their plan. Lastly, they need to *look back* at the completed solution to review and/or discuss its pertinence. Case one lumps the first and second phases of Polya’s problem solving process into one phase, which is the analysis of the problem. For Case One, understanding the problem and devising a plan entails in-depth analysis of what the problem is all about and how to go about it. The rest of the phases are the same. Case One’s problem solving process is cyclical in nature as he “analyze, solve and check” until he arrives at the correct answer. One of the striking differences of Case One’s problem solving process though, is his tendency to seek for help if problems are hard for him to solve. This can be attributed to one of the Filipino’s cultural characteristic referred to as “amoral familism” or lack of individualism. Critiques see it as a Philippine dysfunction where “everyone is expected to look after him or her and his or her immediate family” (http://getrealphilippines.net/). Thus the Filipinos are uncomfortable in new or unstructured situations. Case One manifests this avoidance of uncertainty and lack of individualism by seeking help right away in difficult problem solving situation, even if he has the capacity to hurdle the challenge. The diagram below illustrates the problem solving heuristics of the first case.

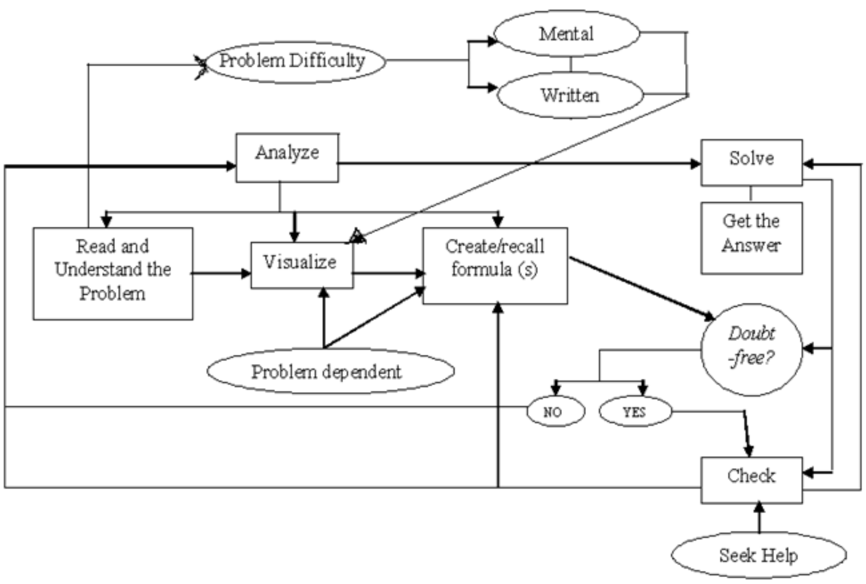


**Figure 1. Problem Solving Heuristics of the First Case**

As mentioned in the preceding section of this paper, Polya (1945) and Krulik and Rudnick (1996) problem solving process has striking similarities and one difference, the existence of the *extend the problem* portion of Krulik and Rudnick's. Due to this reason, Case One's problem solving process is also similar to the Krulik and Rudnick (1996) framework of the problem solving process. Needless to say, Case One's problem solving process conforms more with Polya than with Krulik and Rudnick. The seeking help part was never found on any of them though. The importance of beliefs about problem solving or mathematics in general lies in the assumption that there is some connection between beliefs and behavior (Wilson *et al.,* 1993). The students' personal views about the nature of problem solving clearly defines how they engage in the process. Since Case One believes that one can solve a problem if a formula is already known, he is forced to look for a formula when solving a problem. Outside the taped interview, when asked how he would solve problems that do not require formula, his immediate response is that he uses his common sense. His reliance on formula to solve a problem is very evident in his solutions to problems that require the use of a formula.

**Comparing Incidents to Form Categories (Open Coding)**

The first stage began with open coding. This is the phase where a researcher is tested to trust oneself and in the grounded method, of generating codes and finding relevance as much as possible without preconceived hypothesis. According to Glaser (1998), this process begans with a line-by-line open coding of the data in every way possible. A researcher should ask set of questions like – "What is this data a study of?, " " What category does this incident indicate?," "What is actually happening in the data?," "What is the main concern being faced by the participants?," and "What accounts for the continual resolving of this concern?"These questions kept the researcher theoretically sensitive when analyzing, collecting and coding the data. Table 1 illustrates a sample open coding of an interview transcript.

Focus was on the pattern among incidents that rise above specified description of incidents which yield that code. This is the first level of abstraction drawn from the raw data. Memos were being written while incidents were analyzed. These were done to describe what was happening to the method and the data. The memos themselves captured the relationship between an incident, a coded concept and category. In this study, the researcher wrote memos in detail during the early conduct of the research and this went on into a sentence or two and at other times as the research progresses. In order to validate the emerged concepts and categories at the first open coding and analysis, these memos were also coded in the later part of the research process. This memoing allowed the researcher to move on to the second level of abstraction by constantly comparing incident-to-incident, incident to concept, and concept-to-concept. These were also useful in identifying key points, rather than individual words, and to let the concepts emerge. The selection of points, in order to address the main concern of the participants, is in line with the grounded theory coding analysis and is a protection against data overload (Allan, 2003; Laguda, 2007). Thus, the researcher began to develop theoretical sensitivity, which involves, being able to think in theoretical terms as opposed to quantifying data being done in quantitative data analysis (QDA) research (Laguda, 2007).

**Integrating Categories and their Properties (Axial Coding)**

Understanding the relationships among the categories is the characteristic of this stage. The researcher is like an investigative reporter asking the questions, what, when, where, why and how, and with what result of consequences (Strauss and Corbin, 1998; Laguda, 2007). By answering these questions the researcher intertwines the loose array of concepts and categories. The constant comparative nature of the questions ensures that patterns are not merely woven into two-dimensional pictures of reality, but rather woven into much more complex, three-dimensional constructivist ecology of the participant.

**Table 1. Open Coding of Interview Transcripts**

| Incidents (Translated) | Category/Dimensions |
|---|---|
| Read the problem, Ma'am and under stand what it is all about. | Sense-making Reading the problem |
| If it is not needed in what is asked in the problem, then I will disregard it, but use it if its needed. | Organizing/ Data Analysis/Sorting |
| If there are readily-available formulas, then you can solve it right away. | Belief about the nature of Problem Solving |
| If I already understood the question, know the formula to be used, then I have no doubt that this is the way the problem will be solved. Then I am sure that I had really solved the problem correctly. | Metacognition/ Self-assessment |
| I check Ma'am if I analyze the problem correctly. | Metacognition/ Self-assessment |
| To ensure that it is not wrong….so that it is not prone to errors. | Checking/ Self-monitoring |
| I stop when after re-checking, it is found out to be right. | Self-monitoring |
| If I arrived at a wrong answer then I solve again. | Self-monitoring |
| You need to back understanding the problem. | Self-monitoring/ Whole process |
| I am still thinking, Ma'am, if is it right? | Reflecting/Self-evaluating/ Solution Process |
| Never, Ma'am. (when asked if he gives up on a problem) | Positive Attitude |

**Table 2. Extract from the Sample Conditional Matrix Guide**

| Phenomenon | What | How | Consequences |
|---|---|---|---|
| Sense-making | -what is the problem all about | -translation<br>-reading carefully<br>-repetitive reading<br>-rephrasing | Understanding the problem (partial/complete) |
| Organizing | -what are the data in the problem | -data analysis<br>-sorting data<br>-disregarding useless data<br>-making connections | Understanding the Problem |
| Constructing | -what are the problem information | -making illustrations<br>-visualization<br>-making representations | Understanding the Problem |

Laguda (2007) pointed out that understanding those relationships is not intuitive. In fact, one of the grounded theorists, McCaslin and Scott (2003) suggested developing a Reflective Coding Matrix that would be explained in the third stage. This stage allowed the researcher to be more engaged in effectively understanding the relational dynamics of the categories, and formulate a matrix called a Conditional Relationship Guide (Scott, 2002). The Conditional Relational Guide (CRG) format is designed to ask and answer each relational question about the category named in the far-left column.

- What is [the category]? (using a participant's words/memos helps avoid bias.)
- When does [the category] occur? (Using "during…." helps form the answer.
- Where does [the category] occur? (Using "in…" helps form the answer.)
- Why does [the category] occur? (Using "because…" helps form the answer.)
- How does [the category] occur? Using "by…" helps form the answer.)
- With what Consequences does [the category] occur or is [the category] understood?

Other questions were not used in the CRG of this study because they were found to be inappropriate to be used. The questions on what, how and with what consequences were only used and presented in the CRG in Table 2. This guide attempts to relate the structure to the process. The consequences developed with the guide further contextualized the central phenomenon on the selective coding in the third stage. The consequences were then selected to become substantive categories, i.e., higher-level concepts that emerged from further abstraction of the previous open coding and analysis. Those categories on the guide in the CRG that were not consequences are likely to be dimensions of consequences, and became dimensions on the next stage.

**Delimiting the Theory through Selective Coding and Sorting Categories**

The third stage consisted of selective coding and sorting categories. Also, in this stage, a storyline was developed and formulated. This process involves writing a general descriptive overview, or story line, and verifying it with the data at hand.

The CRG identified the relationships and interactions of the categories one with the others, and described how the consequences of each category were understood. It was at this stage that the researcher primarily focused on the emergence of these key properties and modes of understanding the consequences as an indicator that the study attained theoretical saturation (Glaser, 1978). This was also the beginning of weaving a story line of the many patterns discovered in the CRG. To contextualize the Core Category, the central phenomenon about which all other major and minor categories relate is the primary objective on this stage. Once a Core Category is determined, all other categories become sub-categories. The sub-categories in the relational hierarchy become the Core Category descriptors: the properties, processes, and dimensions.

**Development of Story Line**

The remainder of the selective coding process entailed relating relevant phenomena to the Core Category or the central phenomenon/central concern, always maintaining the central phenomenon at the heart, as an ever widening tapestry as threads of lesser phenomena were tied to and woven around it. The properties and dimensions of the Core Category were more fully developed at this time and the threads of the properties and dimensions of related phenomena, categories and concepts were interlaced and woven tightly together (Laguda, 2007). For the first case of this study, the main story line was:

> *……….how does Case One solve mathematical problems and what are the factors that affect his mathematical problem solving heuristics. Case One's problem solving heuristics is influenced by his beliefs and attitude. The degree of difficulty and the type of the problem determines the computational strategy (mental/written) and the solution strategy (heuristics) he used. Case One's problem solving process is composed of three cyclical and multi-dimensional phases namely: (1) analyzing the problem which includes: (a) understanding the problem through sense-making, organizing and constructing the problem specifics, (b) recalling and determining the strategy or formula to be used in solving; (2) executing the solution strategy selected (solving); and (3) checking if the derived answer is correct or not. If conditions are not met, then the cycle continues until the student is satisfied with his answer. Throughout the process, the use of self-talk, self-questioning, self-monitoring, self-evaluation and self-reflection are very helpful. In other words, Case One used his metacognitive skills throughout the problem solving process. Case One's problem solving heuristics seems to be influenced by his beliefs and attitude. Seeking the help of others in difficult situation is part of his process though. This is a manifestation of a Philippine dysfunction referred to as "amoral familism" or lack of individualism resulting to uncertainty avoidance.*

Using the story line for the first case as a guide, the researcher stepped back again to weave a version of the story at a higher level of abstraction, integrating structure and process in a single statement. Thus, the theory of "Using Metacognitive Skills in Mathematical Problem Solving Heuristics" emerged along the way. Ashman and Conway (1997) characterized *metacognition* as a higher order skill that relates to an individual's awareness of his or her thinking process. According to Yeap (2003, as cited by Garcia, 2004), metacognitive behavior in relation to the problem solving process consists of six categories of basic skills: developing a plan of action, clarifying task requirements, reviewing progress, assessing task difficulty, detecting and using new development, and recognizing errors. Many researchers believed that these metacognitive skills enhance problem solving performance (Garcia, 2004). The most popular proponent of this contention is Alan Schoenfeld. In 1983, he formulated a framework for the analysis of the metacognitive behavior during problem solving. The analysis involves the parsing of problem solving protocols into macroscopic chunks of consistent behavior called episode as follows: reading episode, analysis episode, exploration episode, new information and local assessment, transition, planning-implementation episode and the verification episode. The actual use of Case One's metacognitive skills throughout the problem solving process is evident in this study. It supports the claim that these skills enhance problem solving performance.

**Table 3. Selective Coding for Core Categories**

| |
|---|
| Using Metacognitive Skills in the Mathematical Problem Solving Heuristics" is a theory that has emerged from three multi-distinct yet related main processes of categories:<br>1. analyzing the problem;<br>2. executing the solution strategy,<br>3. checking the problem solving process undertaken,<br>The mathematical problem solving heuristics of Case One has three multi-distinct yet related processes. In each process he frequently asks questions to himself to help him throughout the task. He usually self-reflects, self-assess and self-monitors by asking questions or by self-talk. First, he analyzes the problem. Through analysis of the problem, he can understand the problem and plans for plausible solution strategies. To *understand the problem,* he *makes sense, organizes and constructs* the specifics of the problem. He *makes sense* of the problem through reading or translating statements to dialect. When the degree of difficulty of the problem has been assessed, he chooses to solve it either mentally or in writing depending on the data in the problem. If data in the problems are easy to manipulate he uses mental rather than written computation. He *organizes* the data of the problem through sorting useful and useless data or makes connections when necessary. He *constructs* mental images through *visualization* or *draws* the specifics of the problem in a diagram or a table. He *plans and selects solution strategy* by asking questions like what formula to be used, what to do and so on. Second, he *executes* the strategy he selects from the first phase. Lastly, he *checks* the whole problem solving process he just undertaken by constantly *assessing* and *verifying* his solution and his analysis of the problem. If he finds out that his solution is correct, he finds another solution strategy (easier and shorter) when time is not an issue. If conditions are not met, then he repeats the process until he is satisfied with his answer. He seeks help whenever he finds it difficult to solve the problem alone. The problem solving heuristics of the First Case is influenced by his beliefs and attitude. |

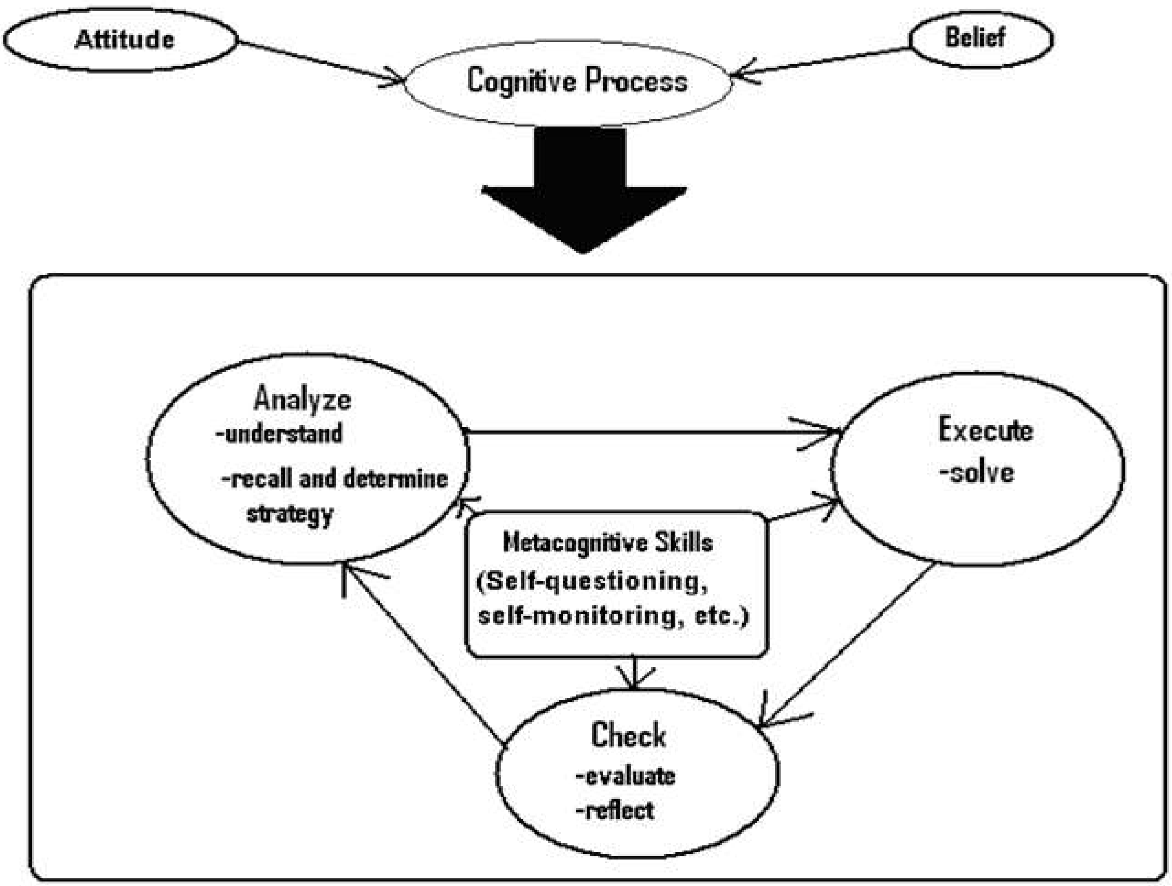


**Figure 2. Initial Framework of the Mathematical Problem Solving Heuristics among Senior High School Students**

Researchs show that good problem solvers actually use these skills with automaticity that make them successful as they are. One concrete example is Case One, who is assessed as good problem solver in school. Richardson (1996) defined attitudes and beliefs as the subset of a group of constructs that name, define, and describe the structure and content of mental states that are thought to drive a person's actions. The effort to promote positive attitudes has been somewhat successful on the individual level. For example, mathematics anxiety can be reduced through systematic desensitization (Hembree, 1990).

On the whole class level the efforts to reform teaching to promote desired attitudes have generally been unsuccessful (McLeod, 1994). However, recent evidence suggests that collaborative approaches can promote positive attitudes among students (e.g. Boaler, 1997a, b, 1998; Ridlon, 1999). Moreover, Maqsud and Khalique (1991) found out that there was a significant positive relationship between mathematics achievement and mathematics attitude (r =.49 for girls; r =.31 for boys). The findings of Ma and Kishor (1997), specifically on the correlation of attitude and achievement were stronger throughout grades 7 to 12, and the findings of Maqsud and Khalique (1991) support the claim that attitude may affect mathematical success of students in problem solving. While Table 3 illustrates the delimiting stage of the theory of the first case through the selective coding process, Figure 2 models the initial framework of the mathematical problem solving heuristics that emerged.

**Writing the Theory**

This is the concluding stage that attempted to explain in writing how the core categories have been developed in elaborating the story line and generating the theory "using metacognitive skills in mathematical problem solving heuristics". Writing the theory includes the rest of the participants in the coding process and utilizes extant literature or theories in each category to strengthen the theory's explanatory power. In grounded theory research, this is called "supplementation" (Gerson, 1991; Laguda, 2007). Gerson (1991, as cited by Laguda, 2007) stated that supplementation is a way of constructing new categories for possible inclusion in developing theory. Conceptually, it lies between coding which names categories and specifies the properties associated with them; and theoretical sampling which tells us what kinds of site or situation we want to look at next. Supplementation starts with extant category, and systematically elaborates contrasting categories in order to provide "raw material" for theoretical sampling, crosscutting and densifying theories, and testing hypotheses. The focus of supplementation is thus on categories not on data; on "might be" rather than "is". The result of supplementation and elaboration is the condition halfway between the beginnings of an inquiry in a situation that is being studied, and its conclusion in a new situation. The succeeding discussion utilized "supplementation and elaboration" to develop the theory "Using Metacognitive Skills in Mathematical Problem Solving Heuristics".

**The Meaning of "Using Metacognitive Skills in the Mathematical Problem Solving Heuristics among Senior High School Students"**

This study was based on a main emergent concern of the participants in the study, which asked how seniors solve mathematical problems. Qualitative method of Pandit (1996), who is a grounded theorist, and the constant comparison method of Glaser and Strauss (1967) were used. Stillman and Galbraith (1998) conducted an intensive study on the problem solving ability of female students at the senior secondary school. The study focused both on the mathematical processing and the underlying cognitive and metacognitive activities that led to that processing. They found out that metacognitive activities were involved in all phases of the solution process with key points in students' solutions identifiable in terms of the cognitive-metacognitve framework of Garofalo and Lester (1985). They described problem solving behavior as consisting of four phases of distinctly different metacognitive activities: orientation, organization, execution, and verification. In describing their framework, they indicated that shifts from one phase to the next commonly occurred when metacognitive decisions resulted in some form of cognitive actions. On the other hand, Carlson and Bloom (2005) in their study about the problem solving behaviors of twelve mathematicians noted that these mathematicians regularly engaged in metacognitive behaviors that involved reflecting on the effectiveness and efficiency of their decisions and actions. These reflections were exhibited frequently during each of the four problem-solving phases (orienting, planning, executing and checking), and they appeared to move the mathematicians' thinking and products in generally productive directions. They recognized that the metacognitve acts within each problem solving phase of the mathematicians were best characterized as acts of monitoring (e.g. reflections on one's thought processes and products). Also, they noted that the mathematicians rarely solved a problem by working through it in linear fashion. These experienced problem solvers typically cycled through the plan-execute-check cycle multiple times when attempting one problem. Sometimes this cycle was slow and tedious; at other times the solver appeared to move through the cycle with little effort. When the checking phase resulted in a rejection of the solution, the solver returned to the planning phase and repeated the cycle. When the checking resulted in an acceptance of the solution, the subject continued to another plan-execute-check cycle until the problem was completed. "Using Metacognitive Skills in Mathematical Problem Solving Heuristics" which emerged from the data, represents the core category or basic cognitive and metacognitive processes by which senior high school students solve mathematical problems. The research findings of Carlson and Bloom (2005) have shown some important bearing on the emergence of this theory.

The theory is now presented in three parts. The first part is the presentation of the coding process of the rest of the participants. The second part is the outline of what is meant by "Using Metacognitive Skills in Mathematical Problem Solving Heuristics". The third part presents a grounded typology for seniors in how they solve mathematical problems using their metacognitive skills. Table 4 illustrates the open coding of the interview transcripts as it appears in the open coding of the interview of all cases. From open coding, memos were made to determine if the coding made was accurate as illustrated in Table 5.

After the coding process of all the cases in the investigation, the data indicate that senior high school students solve mathematical problems "using metacognitive skills" that involve multi-distinct yet related categories and processes, namely: 1) understanding the problem, 2) planning solution strategies, 3) executing the plan, 3) checking the whole process, and 4) reflecting and extending the problem. The success or failure of the students in solving a problem seems to be influenced by some intervening factors such as self-concept, belief, personality, exposure, motivation, attitude, environment, prior knowledge and skills, ability and faith. The problem solving heuristics that emerged from the data is of mixed-nature of Polya (1945), Krulik and Rudnick (1996) problem solving process and Carlson and Bloom (2005) multi-dimensional problem solving framework. The first four phases that emerged in the analysis are actually the four phases of problem solving process by George Polya.

**Table 4. Extract From Open Coding of Interview Transcripts of All Cases**

| Incidents (Translated) | Category/ Sub-category/ Dimensions |
|---|---|
| I will look for the given that I can use and what is asked in the problem. | Identifying/ Problem Information |
| Uhm, I understand it in English. | Understanding/ Problem Information |
| …I will first use the given that is easy to solve then proceed to the difficult one. | Prioritizing/ Degree of difficulty |
| I analyze the problem, Ma'am. Then I use every data given. If questions are asked directly, and then I will answer them directly, too. Then I will also apply it to all the data which I used, Ma'am. | Analyzing/ Given data/ information |
| There are problems, Ma'am, that we can't use all the given data. | Disregarding/ Sorting/ Un-useful/ Useful data or information |
| I imagine them, Ma'am. | Imagining/ Visualizing/ Problem Information |
| I think about it….. | Thinking/ Problem Information |
| ...uhmm...I will ….as if I am the one in the position given in the problem…. | Imagining/ Putting oneself in a problem situation |
| What operations are to be used? | Identifying/ Task requirement |
| The problem? The question? Are they related to the given? Are they connected? | Analyzing (through asking questions to one self)/ Problem Information |
| Is my answer correct? Did I reduce it in the simplest form? | Self-evaluating/ Derived Answer |
| Can it be answered/solved in other way? | Evaluating/ Another plan of action/ problem solution |
| Are there any other formulas to be used? | Checking/ Developing/ Another plan of action/ Problem solution |
| I understand the problem if I feel that my answer is already correct. | Understanding/ Emotionally-based |
| I understand the problem if nothing is bothering in my mind. | Understanding/ Rationally-based |
| Usually, I draw, Ma'am. If I make some graph, then it is just a graph. For instance I already started to draw, then I do some charting. But when I solve, sometimes I did not draw anymore, you can't use every strategy. ..If there are lots of things to solve, then I solve them slowly…step-by-step. | Visualizing/ Conjecturing/ Selecting Strategy/ Problem-dependent |
| I solve it slowly in my mind…I use my mind/memory…there are problems that are easy to think about…you can even answer them directly. | Computational Strategy/ (Mental/written)/ Problem-dependent |

**Table 5. Extract from the First Open Coded Memos**

| Memos | Code/categories/subcategories/properties |
|---|---|
| 1. Understanding the problem may be made possible through reading the problem repeatedly. | Understanding through Repetitive Reading |
| 2. Translating the problem to the dialect facilitates understanding of the problem. | Understanding through translation |
| 3. Problems have varying degree of difficulty. | Problem difficulty |
| 4. The degree of difficulty of the problems makes respondents choose either to solve them mentally or in writing. | Problem Solving Strategy (Technical) |
| 5. Easy problems with smaller numbers may be solved mentally. | Mental computation (Condition) |
| 6. Problems involving larger numbers may be solved in writing or with the use of technology depending on students' preference. | Written Computation (Condition) |
| 7. The choice of strategy in solving depends on the type of problem to be solved. | Problem Dependent Strategy |
| 8. Problems may be solved following the procedures set by the teacher. | Teacher-directed Strategy |
| 9. Problems may be solved through the students' usual problem solving strategy they are accustomed with. | Personal-preference Strategy/personal choice |
| 10. Making connections on the different problem parts and data depends on the problem-type. | Problem-dependent Action |
| 11. Illustrating or drawing the specifics of the problem is one way of understanding it. | Constructing (problem specifics) |
| 12. Asking questions to oneself facilitates comprehension of what the problem is all about. | Self-talk and Self-questioning |

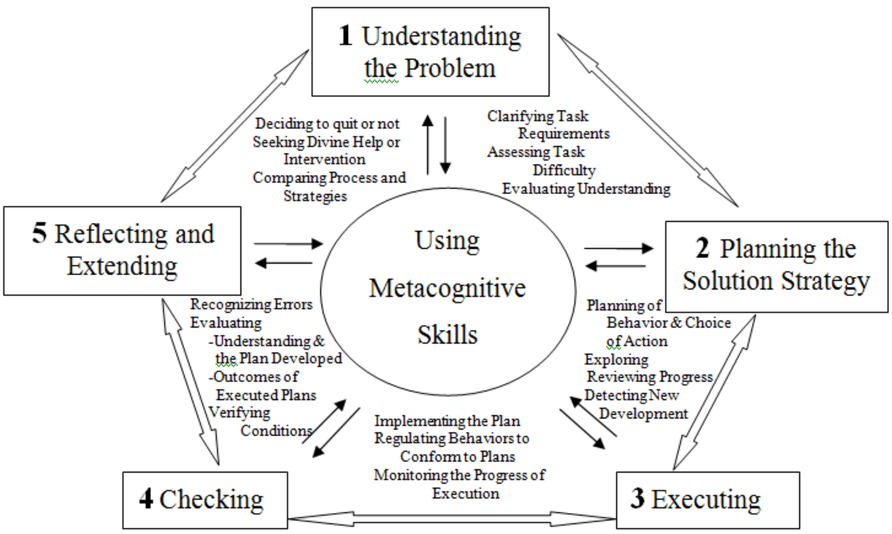


**Figure 3. Emerged Integrative Construct of Core Categories**

The remaining fifth phase is actually the last phase of the problem solving process by Krulik and Rudnick (but with striking difference). Although the five multi-distinct yet related phases or processes are similar to that of George Polya and Krulik and Rudnick, there exists a striking difference between and among them. In this study, each process or phase (excluding executing) has sub-processes which are actually affected by ones metacognitive skills. Any problem solver may either skip and jump from one phase to another or go back to each phase in cyclical manner or with flexibility depending upon their creativity. The study of Carlson and Bloom (2005) describes the problem solving behaviors of 12 mathematicians as they completed four mathematical tasks. The emergent problem solving framework that they had drawn on the large body of research, was grounded by and modified in response to close observations of the mathematicians. They had developed the Multidimensional Problem Solving Framework which is composed of four phases: orientation, planning, executing and checking. The study of Polya (1945), Krulik and Rudnick (1996), and Carlson and Bloom (2005), have important bearing on the present study.

Their similarities and difference with the present investigation are described and presented as categories and sub-categories are illustrated in the succeeding paragraphs. Figure 3 illustrates the emerged integrative construct of the core categories of using metacognitive skills in the mathematical problem solving among senior high school students. The central construct that emerged from the investigation was the students' use of metacognitive skills in mathematical problem solving. Five main processes were identified as encompassing an emerging substantive theory on the students' use of metacognitive skills. The multi-distinct yet related processes are: 1) *understanding* the problem through *sense-making, organizing* and *constructing* useful information from the problem, 2) *planning* solution strategies by *identifying, conjecturing* and *selecting* strategies, 3) *executing* the plan, 4) *checking* the process and strategies undertaken, and 5) *reflecting and extending* the problem. In each process, students frequently ask themselves certain questions. Most of the time, they self-direct, self-question through self-talk, self-monitor and self-evaluate throughout the problem solving process.

The study revealed that the multi-distinct yet related processes could serve as a grounded frame of reference in which students do mathematical problem solving. Some students find themselves incapable to proceed to the next phase, causing them to give up. Others just try to come up with meaningless computations. Some prefer to use teacher-introduced solution strategies, others use trial-and-error or resort to guessing. Others ask for Divine help and the help of other people when they are stuck in doing the task. This last attribute is a manifestation of a Filipino dysfunction referred to as "amoral familism" or lack of individualism. Consequently, Filipinos also avoid uncertain or ambiguous situations and tend to be strongly religious (http://getrealphilippinesnet/). The problem solving process is neither linear nor cyclical. It is flexible since one moves from one phase to the next then goes back to the first phase depending on the problem requirement and one's satisfaction. Problem solvers go through the process with varying degrees of creativity and flexibility depending on such factors as the problem at hand, students' metacognitive skills, and prior knowledge.

The emergent theory proposes that certain intervening factors such as self-concept, belief, personality, exposure, motivation, attitude, environment, prior knowledge and skills, ability and faith could have influence in the multi-distinct processes of using metacognitive skills in mathematical problem solving. This study captures all the findings culled from the data in an emergent model on the processes Filipino students undergo, and the heuristics they use during mathematical problem solving. The model highlights the role that their metacognitive skills and other intervening factors play in their success or failure to do the task. This model can be further refined in an extended discussion. At this point, the study has generated a substantive theory on the area of interest of the researchers. There is no claim of applicability of the model for all Filipino students.

### Implications for Practice

Aside from building a theory grounded on the gathered data, the generated model on "using metacognitive skills in problem solving heuristics" has implications for practice in Philippine education. It points out to the significant role that metacognitive skills play in the problem solving heuristics of secondary students, and to the cultural attributes of the Filipino students that hamper the development of independent problem solving skills. With the help of the model, school administrators, curriculum makers and teachers, can be guided on how to provide opportunities for students to develop their metacognitive skills. This can be in the form of a curriculum that promotes problem based learning, a pedagogy that uses problem solving in various ways or a learning environment that supports the development of students' problem solving skills and independent learning skills.The theory can also serve as a framework in which the students can better understand the role their metacognitive skills play in the problem solving process and in their mathematical performance as well. The theory can encourage Filipino students to gain metacognitive knowledge in order to become successful problem solvers and independent lifelong learners.

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