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PROBABILITY DISTRIBUTIONS OF WIND SPEED HISTORICAL DATA SERIES IN MOSSORÓ, NORTHEASTERN BRAZIL

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ABSTRACT

We studied seven probability distributions (Normal, Log-Normal, Beta, Gamma, Log-Pearson Type III, Gumbel, and Weibull) to verify their adjustment to historical data series of average monthly wind speed in the municipality of Mossoró, northeastern Brazil. To verify the goodness of fit of data to these models, we applied the Maximum Likelihood Logarithm and the Kolmogorov-Smirnov, Kuiper, Cramer-von Mises, Anderson-Darling, and Chi-squared tests at a significance level of 10%, totaling 504 tests. The Gumbel probability distribution provided the best fit for the data of monthly wind speed. All the criteria used to verify the goodness of fit were concordant, except the Maximum Likelihood Logarithm for some adjusted periods and models.

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INTRODUCTION

Statistical techniques applied to meteorological data allows condensing the massive amount of data recorded in stations into simple tables or equations, summarizing all the information to simplify the inferences about the data (Assis *et al.*, 1996). Important decisions about agricultural activities and investments on clean sources of energy require knowledge about the local patterns of rainfall, temperature, relative air humidity, evaporation, solar radiation, the occurrence of dew, fog, hail, frost, snow, and wind direction and speed, among others. A theoretical distribution is an abstract mathematical form or particular format. Some of these mathematical forms appear naturally as a consequence of specific data-generating processes, and when applicable, they are plausible candidates for concisely represent variations in a set of data.

Even when there is no substantial natural basis behind the choice of a particular theoretical distribution, one can empirically find that some distribution model represents a set of data very well. The nature of data directly guides the choice of the probability density functions. Some functions show reasonable estimation capacity for small numbers of data, and others require a large number of observations. Due to the number of parameters of their equations, some density functions can assume different forms, fitting into a high number of situations, that is, they are flexible (Catalunha *et al.*, 2002). The statistical distribution of wind speeds is an essential tool for the evaluation of wind energy potential and its performance in energy conversion systems. There are several functions of probability distributions for random variables that adjust to continuous data, for example, the Uniform, Normal, Log-Normal, Gamma, Extreme Values or Gumbel, Weibull, Exponential, and Beta Distributions. Several studies investigated the fit of probability density distributions to historical data series of climatic variables such as air temperature (Burio *et al.*, 2000A; Burio *et al.*, 2000B; Assis

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et al., 2004), wind speed (Van Der Auwera *et al.* 1980; Garcia *et al.*, 1998; Bautista, 2002; Sansigolo, 2005; Sansigolo, 2008), and solar radiation (Buriol *et al.*, 2000A; Assis *et al.*, 2004). After proving the goodness of fit of the data to the theoretical distribution, the adjusted models can provide useful information for the planning of several activities (Fietz *et al.*, 1997). The goodness of fit tests such as Chi-squared, Kolmogorov-Smirnov, and Cramer-von Mises (Campos, 1983) compare the empirical probabilities of a variable with the theoretical probabilities estimated by the distribution function. They verify whether the sample values come from a population with a particular theoretical distribution. The Chi-squared and Kolmogorov-Smirnov tests are widely used (Assis *et al.*, 1996), the former being usually more efficient than the latter (Catalunha *et al.*, 2002). The tests of Cramer-von Mises and Anderson-Darling also differ in their efficacy (Campos, 1983). Given the above, the purpose of this study was to assess the fit of the monthly average wind speed series in Mossoró, northeastern Brazil, to the following probability density functions: Normal, Log-Normal, Beta, Gamma, Log-Pearson Type III, Gumbel, and Weibull. Our results enable us to make reliable predictions of the local wind speed for any requested percentage, as well as estimating monthly wind speed in the region.

MATERIALS AND METHODS

The wind speed data (m.s^{-1}) were acquired from a series of 38 years (from 1970 to 2007) recorded by the Federal Meteorological Station in Mossoró, UFERSA ($5^{\circ}11'\text{S}$ – $37^{\circ}20'\text{W}$; 18 m of altitude). The region has a mean annual temperature of 27.5°C and relative humidity of 68.9% (Carmo Filho *et al.*, 1991). According to Köppen's climatic classification, the climate of Mossoró is BSwh', that is, hot and dry. The average wind speed was measured at a 10 meters height station. In each month of the year, the data of the series were adjusted to seven models of probability distributions: Normal, Log-Normal, Beta, Range, Log-Pearson Type III, Gumbel, and Weibull.

To verify the fit of data to the models, we applied both the Maximum Likelihood Logarithm ($\ln(L)$) and the following goodness of fit tests: Kolmogorov-Smirnov (KS), Kuiper (K), Cramer-von Mises (CVM), Anderson-Darling (AD), and Chi-Squared (χ^2). A total of 504 series of average wind speed data were analyzed using the software VTFIT (Campos, 1983; Assis *et al.*, 1996; Cooke, 1993). We used the numerical value of the Logarithm of Maximum Likelihood function $\ln(L) = \sum_{i=1}^n \ln[\hat{f}(x_i)]$ to indicate the degree of adjustment

(Worley *et al.*, 1990; Cooke, 1993). This statistic can be a negative number, and it has the power to discriminate settings in which the other tests fail to point (Shapiro; Brain, 1981). The higher the logarithm of the maximum likelihood function, the better the adjustment obtained.

RESULTS AND DISCUSSION

The Gumbel probability distribution provided the best fit for the data of monthly wind speed (Tables 1 to 14). However, all studied functions adjusted well to the data (Normal, Log-Normal, Beta, Gamma, Log-Pearson Type III, Gumbel, and Weibull). Our results suggest that the Gumbel distribution can be used to estimate monthly wind speed in Mossoró and surrounding regions, as well as to predict the occurrence of meteorological events. This tool may guide decisions, planning, and forecasting for all activities dependent on wind speed, for example, the installation of wind power plants. Small differences in the value of the average monthly wind speed led to some divergences in the results obtained by the various fitting criteria. The results of the Maximum Likelihood Logarithm indicated some non-significant adjustments (Tables 1 to 12), which may be caused by the convergence of adjusted models and the nature of the data (Hollander; Wolfe, 1999; Carneiro *et al.*, 2001; Catalunha *et al.*, 2002). On the other hand, the Chi-squared and Kolmogorov-Smirnov tests indicated a better goodness of fit than the Maximum Likelihood Logarithm (Table 13).

Table 1 - Criteria for the goodness of fit of wind speed historical data series to seven theoretical distributions of probability Months of January from 1970 to 2007, Mossoró, northeastern Brazil

Distribution probability	Goodness of Fit					
	Maximum Likelihood Logarithm ($\ln(L)$)	Kolmogorov-Smirnov (KS)	Kuiper (K)	Cramer-von Mises	Anderson-Darling (AD)	Chi-Squared (χ^2)
Normal	-7.912	0.127	0.222	0.404	0.048	0.309
Log-Normal	-7.489	0.122	0.213	0.357	0.044	0.309
Beta	3.332	0.164	0.306	5.234	0.160	0.309
Gama	-7.688	0.114	0.195	0.351	0.042	0.309
Log-Pearson III	-6.976	0.111	0.212	0.322	0.044	0.309
Gumbel	-12.214	0.147	0.287	1.060	0.135	0.273
Weibull	-11.075	0.135	0.260	0.866	0.106	0.273

Table 2 - Criteria for the goodness of fit of wind speed historical data series to seven theoretical distributions of probability Months of February from 1970 to 2007, Mossoró, northeastern Brazil

Distribution probability	Goodness of Fit					
	Maximum Likelihood Logarithm ($\ln(L)$)	Kolmogorov-Smirnov (KS)	Kuiper (K)	Cramer-von Mises	Anderson-Darling (AD)	Chi-Squared (χ^2)
Normal	-18.897	0.127	0.518	0.560	0.075	0.518
Log-Normal	-19.861	0.137	0.239	0.625	0.072	0.518
Gama	-8.653	0.157	0.300	2.677	0.143	0.199
Beta	-19.511	0.147	0.260	0.657	0.083	0.518
Log-Pearson III	-19.273	0.139	0.273	0.648	0.093	0.291
Gumbel	-21.270	0.170	0.339	0.992	0.156	0.038
Weibull	-19.853	0.153	0.306	0.773	0.120	0.291

Table 3. Criteria for the goodness of fit of wind speed historical data series to seven theoretical distributions of probability Months of March from 1970 to 2007, Mossoró, northeastern Brazil

Distribution probability	Goodness of Fit					
	Maximum Likelihood Logarithm (ln(L))	Kolmogorov-Smirnov (KS)	Kuiper (K)	Cramer-von Mises	Anderson-Darling (AD)	Chi-Squared (χ^2)
Normal	-14.473	0.141	0.277	0.837	0.116	0.660
Log-Normal	-14.520	0.138	0.268	0.816	0.116	0.349
Gama	-4.187	0.205	0.377	6.745	0.208	0.241
Beta	-14.542	0.126	0.247	0.693	0.092	0.660
Log-Pearson III	-14.326	0.135	0.265	0.758	0.103	0.660
Gumbel	-15.153	0.147	0.285	0.961	0.134	0.491
Weibull	-14.635	0.144	0.282	0.893	0.124	0.491

Table 4 - Criteria for the goodness of fit of wind speed historical data series to seven theoretical distributions of probability Months of April from 1970 to 2007, Mossoró, northeastern Brazil

Distribution probability	Goodness of Fit					
	Maximum Likelihood Logarithm (ln(L))	Kolmogorov-Smirnov (KS)	Kuiper (K)	Cramer-von Mises	Anderson-Darling (AD)	Chi-Squared (χ^2)
Normal	3.764	0.167	0.316	0.715	0.113	0.731
Log-Normal	3.819	0.159	0.313	0.703	0.110	0.731
Beta	11.130	0.210	0.386	5.629	0.229	0.731
Gama	3.740	0.150	0.292	0.600	0.090	0.731
Log-Pearson III	3.810	0.158	0.304	0.648	0.100	0.731
Gumbel	2.486	0.178	0.324	0.817	0.127	0.731
Weibull	3.017	0.178	0.322	0.766	0.120	0.731

Table 5 - Criteria for the goodness of fit of wind speed historical data series to seven theoretical distributions of probability Months of May from 1970 to 2007, Mossoró, northeastern Brazil

Distribution probability	Goodness of Fit					
	Maximum Likelihood Logarithm (ln(L))	Kolmogorov-Smirnov (KS)	Kuiper (K)	Cramer-von Mises	Anderson-Darling (AD)	Chi-Squared (χ^2)
Normal	0.112	0.120	0.225	0.468	0.067	0.779
Log-Normal	0.351	0.111	0.220	0.451	0.063	0.779
Beta	6.558	0.157	0.282	3.883	0.106	0.779
Gama	0.220	0.779	0.200	0.385	0.051	0.779
Log-Pearson III	0.348	0.109	0.216	0.429	0.059	0.779
Gumbel	-2.364	0.143	0.254	0.696	0.102	0.349
Weibull	-1.425	0.139	0.245	0.588	0.085	0.349

Table 6 - Criteria for the goodness of fit of wind speed historical data series to seven theoretical distributions of probability Months of June from 1970 to 2007, Mossoró, northeastern Brazil

Distribution probability	Goodness of Fit					
	Maximum Likelihood Logarithm (ln(L))	Kolmogorov-Smirnov (KS)	Kuiper (K)	Cramer-von Mises	Anderson-Darling (AD)	Chi-Squared (χ^2)
Normal	3.046	0.191	0.348	0.772	0.134	0.269
Log-Normal	3.345	0.176	0.333	0.732	0.129	0.269
Beta	9.665	0.238	0.443	5.197	0.235	0.025
Gama	3.193	0.182	0.333	0.743	0.138	0.269
Log-Pearson III	3.381	0.174	0.333	0.747	0.133	0.269
Gumbel	-0.877	0.240	0.447	1.477	0.256	0.001
Weibull	0.375	0.230	0.426	1.246	0.216	0.007

Table 7 - Criteria for the goodness of fit of wind speed historical data series to seven theoretical distributions of probability Months of July from 1970 to 2007, Mossoró, northeastern Brazil

Distribution probability	Goodness of Fit					
	Maximum Likelihood Logarithm (ln(L))	Kolmogorov-Smirnov (KS)	Kuiper (K)	Cramer-von Mises	Anderson-Darling (AD)	Chi-Squared (χ^2)
Normal	-8.393	0.145	0.258	0.587	0.104	0.141
Log-Normal	-8.082	0.127	0.242	0.491	0.083	0.141
Beta	-4.073	0.143	0.279	2.515	0.131	0.080
Gama	-8.215	0.131	0.256	0.544	0.097	0.141
Log-Pearson III	-8.176	0.123	0.236	0.470	0.076	0.836
Gumbel	-12.566	0.178	0.324	1.347	0.233	0.060
Weibull	-10.988	0.170	0.307	1.075	0.188	0.060

Table 8 - Criteria for the goodness of fit of wind speed historical data series to seven theoretical distributions of probability
Months of August from 1970 to 2007, Mossoró, northeastern Brazil

Distribution probability	Goodness of Fit					
	Maximum Likelihood Logarithm (ln(L))	Kolmogorov-Smirnov (KS)	Kuiper (K)	Cramer-von Mises	Anderson-Darling (AD)	Chi-Squared (χ^2)
Normal	-8.393	0.117	0.225	0.409	0.060	0.602
Log-Normal	-8.304	0.122	0.233	0.419	0.065	0.602
Beta	-1.261	0.143	0.254	3.903	0.099	0.602
Gama	-8.392	0.114	0.215	0.352	0.051	0.602
Log-Pearson III	-8.361	0.116	0.219	0.362	0.054	0.602
Gumbel	-10.472	0.124	0.225	0.574	0.073	0.080
Weibull	-9.732	0.112	0.200	0.490	0.063	0.080

Table 9 - Criteria for the goodness of fit of wind speed historical data series to seven theoretical distributions of probability.
Months of September from 1970 to 2007, Mossoró, northeastern Brazil

Distribution probability	Goodness of Fit					
	Maximum Likelihood Logarithm (ln(L))	Kolmogorov-Smirnov (KS)	Kuiper (K)	Cramer-von Mises	Anderson-Darling (AD)	Chi-Squared (χ^2)
Normal	-16.644	0.154	0.274	0.559	0.081	0.392
Log-Normal	-6.946	0.154	0.316	4.020	0.128	0.392
Beta	-16.523	0.159	0.252	0.494	0.072	0.608
Gama	-15.305	0.143	0.268	0.371	0.060	0.392
Log-Pearson III	-15.719	0.138	0.291	0.438	0.075	0.731
Gumbel	-15.489	0.160	0.290	0.422	0.072	0.945
Weibull	11.029	0.150	0.290	0.701	0.113	0.731

Table 10 - Criteria for the goodness of fit of wind speed historical data series to seven theoretical distributions of probability
Months of October from 1970 to 2007, Mossoró, northeastern Brazil

Distribution probability	Goodness of Fit					
	Maximum Likelihood Logarithm (ln(L))	Kolmogorov-Smirnov (KS)	Kuiper (K)	Cramer-von Mises	Anderson-Darling (AD)	Chi-Squared (χ^2)
Normal	10.766	0.158	0.296	0.731	0.116	0.123
Log-Normal	16.349	0.164	0.296	2.669	0.123	0.349
Beta	10.779	0.164	0.307	0.741	0.121	0.123
Gama	10.779	0.169	0.307	0.741	0.121	0.349
Log-Pearson III	11.430	0.164	0.296	0.715	0.125	0.123
Gumbel	10.891	0.166	0.300	0.787	0.140	0.002
Weibull	11.099	0.165	0.298	0.755	0.134	0.002

Table 11 - Criteria for the goodness of fit of wind speed historical data series to seven theoretical distributions of probability.
Months of November from 1970 to 2007, Mossoró, northeastern Brazil

Distribution probability	Goodness of Fit					
	Maximum Likelihood Logarithm (ln(L))	Kolmogorov-Smirnov (KS)	Kuiper (K)	Cramer-von Mises	Anderson-Darling (AD)	Chi-Squared (χ^2)
Normal	-1.128	0.211	0.388	1.563	0.238	0.731
Log-Normal	0.314	0.199	0.364	1.333	0.201	0.731
Beta	14.447	0.137	0.250	2.435	0.097	0.269
Gama	-0.229	0.207	0.381	1.529	0.236	0.055
Log-Pearson III	-1104.349	0.367	0.700	4.592	0.805	0.002
Gumbel	-13.135	0.272	0.531	3.963	0.695	0.000
Weibull	-10.959	0.257	0.505	3.601	0.621	0.000

Table 12 - Criteria for the goodness of fit of wind speed historical data series to seven theoretical distributions of probability.
Months of December from 1970 to 2007, Mossoró, northeastern Brazil

Distribution probability	Goodness of Fit					
	Maximum Likelihood Logarithm (ln(L))	Kolmogorov-Smirnov (KS)	Kuiper (K)	Cramer-von Mises	Anderson-Darling (AD)	Chi-Squared (χ^2)
Normal	-6.406	0.100	0.197	0.271	0.043	0.602
Log-Normal	-6.709	0.110	0.207	0.312	0.049	0.309
Beta	-0.694	0.120	0.199	2.196	0.050	0.602
Gama	-6.668	0.109	0.211	0.301	0.048	0.309
Log-Pearson III	-6.134	0.122	0.194	0.246	0.039	0.602
Gumbel	-7.122	0.115	0.197	0.413	0.062	0.602
Weibull	-6.714	0.113	0.194	0.345	0.052	0.602

Table 13 - The p-values for the fit of monthly wind speed series from 1970 to 2007 in Mossoró, northeastern Brazil, to seven distribution probability functions

Date of sowing	Probability density distributions													
	Normal		Log-Normal		Beta		Gama		Log-Pearson Tipo III		Gumbel		Weibull	
	P-value KS	P-value χ^2	P-value KS	P-value χ^2	P-value KS	P-value χ^2	P-value KS	P-value χ^2	P-value KS	P-value χ^2	P-value KS	P-value χ^2	P-value KS	P-value χ^2
January	0.127	0.309	0.122	0.309	0.164	0.309	0.114	0.309	0.111	0.309	0.147	0.273	0.135	0.273
February	0.127	0.518	0.137	0.518	0.157	0.199	0.147	0.518	0.139	0.291	0.170	0.038	0.153	0.291
March	0.141	0.660	0.138	0.349	0.205	0.241	0.126	0.660	0.135	0.660	0.147	0.491	0.144	0.491
April	0.167	0.731	0.159	0.731	0.210	0.731	0.150	0.731	0.158	0.731	0.178	0.731	0.178	0.731
May	0.120	0.779	0.111	0.779	0.157	0.779	0.779	0.779	0.109	0.779	0.143	0.349	0.139	0.349
June	0.191	0.269	0.176	0.269	0.238	0.025	0.182	0.269	0.174	0.269	0.240	0.001	0.230	0.007
July	0.145	0.141	0.127	0.141	0.143	0.080	0.131	0.141	0.123	0.836	0.178	0.060	0.170	0.060
August	0.117	0.602	0.122	0.602	0.143	0.602	0.114	0.602	0.116	0.602	0.124	0.080	0.112	0.080
September	0.154	0.392	0.154	0.392	0.159	0.608	0.143	0.392	0.138	0.731	0.160	0.945	0.150	0.731
October	0.158	0.123	0.164	0.349	0.164	0.123	0.169	0.349	0.164	0.123	0.166	0.002	0.165	0.002
November	0.211	0.731	0.199	0.731	0.137	0.269	0.207	0.055	0.367	0.002	0.272	0.000	0.257	0.000
December	0.110	0.602	0.120	0.309	0.109	0.602	0.122	0.309	0.100	0.602	0.115	0.602	0.113	0.602

Table 14 - Fitting frequencies of wind speed historical data series from January to December of 1970 to 2007 to seven theoretical distribution models of probability densities. Mossoró, northeastern Brazil

Date of sowing	Probability density distributions													
	Normal		Log-Normal		Beta		Gama		Log-Pearson Tipo III		Gumbel		Weibull	
	P-value KS	P-value χ^2	P-value KS	P-value χ^2	P-value KS	P-value χ^2	P-value KS	P-value χ^2	P-value KS	P-value χ^2	P-value KS	P-value χ^2	P-value KS	P-value χ^2
January	F	F	F	F	F	F	F	F	F	F	F	F	F	F
February	F	F	F	F	F	F	F	F	F	F	-	F	F	F
March	F	F	F	F	F	F	F	F	F	F	F	F	F	F
April	F	F	F	F	F	F	F	F	F	F	F	F	F	F
May	F	F	F	F	F	F	F	F	F	F	F	F	F	F
June	F	F	F	F	F	-	F	F	F	F	F	-	F	-
July	F	F	F	F	F	-	F	F	F	F	F	-	F	-
August	F	F	F	F	F	F	F	F	F	F	-	F	-	
September	F	F	F	F	F	F	F	F	F	F	F	F	F	F
October	F	F	F	F	F	F	F	F	F	F	-	F	-	
November	F	F	F	F	F	F	F	-	F	-	F	-	F	-
December	F	F	F	F	F	F	F	F	F	F	F	F	F	F

(F = Fitted distribution. “-” = Non-fitted distribution (10% of probability).)

We may explain such results by the high level of approval of distributions by the Kolmogorov-Smirnov test. The Chi-squared test was somewhat more rigorous than the Kolmogorov-Smirnov because the Chi-squared requires a more extensive historical data series and the grouping of data into classes of frequencies (Hollander; Wolfe, 1999; Carneiro et al., 2001; Catalunha et al., 2002). The Cramer-von Mises, Anderson-Darling, and Kuiper distributions showed strong similarities in the fit. In these cases the nonparametric evidence of significance does not require a minimum size of historical data series nor the grouping of data into frequency classes (Siegel, 1977; Campos, 1983; Hollander; Wolfe, 1999; Carneiro et al., 2001; Catalunha et al., 2002).

Conclusion

The Gumbel probability distribution provided the best fit for the data of monthly wind speed. The data also fitted to Normal, Log-Normal, Beta, Gamma, Log-Pearson Type III, and Weibull probability distribution functions. All the criteria used to verify the goodness of fit were concordant, except the Maximum Likelihood Logarithm for some periods and models.

REFERENCES

- Assad, E.D., Castro, L.H.R. Análise freqüencial da pluviometria para a estação de Sete Lagoas. Pesquisa Agropecuária Brasileira, v.26, n.3, p.397-402, 1991.
- Assis, F. N., Arruda, H. V. De, PEREIRA, A. R. Aplicações de estatística à climatologia: teoria e prática. Pelotas: UFPEL, 1996. 161p.
- Assis, F.N. Modelagem da ocorrência da quantidade de chuva e de dias secos em Piracicaba (SP) e Pelotas-RS. Piracicaba. 1991. 134p. (Doutorado) - Escola Superior de Agricultura "Luiz de Queiroz", Universidade de São Paulo.
- Assis, J. P., Neto, D. D., Manfron, P. A., Martin, T. N., Sparovek, G., Timm, L. C.. Ajuste de séries históricas de temperatura e radiação solar global às funções densidade de probabilidade normal e log-normal, em Piracicaba, SP. Revista Brasileira de Agrometeorologia, Santa Maria, v. 12, n. 1, p. 113-121, 2004.
- Ávila, A. M. H., Berlato, M. A., Silva, J. B., Fontana, D. C. Probabilidade de ocorrência de precipitação pluvial mensal igual ou maior que a evapotranspiração potencial para a estação de crescimento das culturas de primavera-verão no Estado do Rio Grande do Sul. Pesquisa Agropecuária Gaúcha, Porto Alegre, v.2 p. 149-154, 1996.
- Bautista, E. Z. A distribuição generalizada de valores extremos no estudo da velocidade máxima do vento em Piracicaba, SP. Dissertação de Mestrado, ESALQ, USP, Piracicaba, 47p. 2002.
- Berlato, M. A. Modelo de relação entre o rendimento de grãos de soja e o déficit hídrico para o Estado do Rio Grande do Sul. 1987. 93 f. Tese (Doutorado) – Instituto Nacional de Pesquisas Espaciais, São José dos Campos, 1987.
- Botelho, V.A., MORAIS, A.R. Estimativas dos parâmetros da distribuição gama de dados pluviométricos do município de Lavras, Estado de Minas Gerais. Ciências e Agrotecnologia, v.23, n.3, p.697-706, 1999.
- Buriol, G. A., Estefanel, V., Andriolo, J. L., Matzenauer, R., Marcon, I. A. Condições térmicas para o cultivo do pepineiro na região do baixo vale do Taquari, RS: 1. temperaturas baixas limitantes. Pesquisa Agropecuária Gaúcha, Porto Alegre, v. 6, p. 205-213, 2000B.
- Buriol, G.A., Heldwein, A.B., Estefanel, V., Matzenauer, R., Marcon, I.A. Disponibilidade de radiação solar para o cultivo do tomateiro durante o inverno no Estado do Rio grande do Sul. Pesquisa Agropecuária Gaúcha, v.6, p.113-120, 2000A.
- Campos, H. de. Estatística experimental não-paramétrica. 4 ed. Piracicaba: 4^a ed. ESALQ, 1983. 349 p.
- Carmo Filho, F., Espínola Sobrinho, J., Maia Neto, J. M. Dados meteorológicos de Mossoró (janeiro de 1989 a dezembro de 1990). Mossoró: ESAM, FGD, 1991. 110p. Coleção Mossoroense, Série C, 630.
- Carneiro, J. W. P., Guedes, T. A., Amaral, D. Descrição do tamanho de sementes de milho em lotes disponíveis no comércio. Revista Brasileira de Sementes. Vol. 23, nº 2, p. 209-218, 2001.
- Castro, R. Distribuição probabilística de precipitação na região de Botucatu – SP. 1996. 88 p. Dissertação (Mestrado em Agronomia) – Universidade Estadual Paulista, Botucatu, 1996.
- Catalunha, M.J., Sediyama, G.C., Leal, B.G., Soares, C.P.B., Ribeiro, A. Aplicação de cinco funções densidade de probabilidade a séries de precipitação pluvial no Estado de Minas Gerais. Revista Brasileira de Agrometeorologia, v.10, n.1, p.153-162, 2002.
- Cooke, R.A. Vtfit: A routine for fitting homogenous Probability density functions. User documentation. Blacksburg: Department of Agricultural Engineering, Virginia, Polytechnic Institute, 1993. 21p.
- Dourado Neto, D., Assis, J. P., Timm, L. C., Manfron, P. A., Sparovek, G., Martin, T. N. Ajuste de modelos de distribuição de probabilidade a séries históricas de precipitação pluvial diária em Piracicaba-SP. Revista Brasileira de Agrometeorologia, Santa Maria, v. 13, n. 2, p.273-283, 2005.
- Fietz, C.R., Frizzone, J.A., Folegatti, M.V., Pinto, J.M. Probabilidade de ocorrência da evapotranspiração de referência na região de Dourados, MS. Ciência Rural, Santa Maria, v. 27, n. 2, p.207-210, 1997.
- Garcia, A., Torres, J. L., Prieto, E., Francisco, A. Fitting Wind Speed Distributions: A Case Study. Solar Energy, v. 6, n. 2, p. 139-144, 1998.
- Hollander, M., Wolfe, D.A. Nonparametric statistical methods. 2nd ed, John Wiley & Sons, New York, 1999.
- Sansigolo, C. A. Distribuições de probabilidade de velocidade e potência do vento. Revista Brasileira de Meteorologia. v. 20, n. 2, 207-214, 2005.
- Sansigolo, C.A. Distribuições de extremos de precipitação diária, temperatura máxima e mínima e velocidade do vento em Piracicaba, SP (1917-2006). Revista Brasileira de Meteorologia, v. 23, n. 3, p. 341-346, 2008.
- Shapiro, S. S., Brain, C. W. A review of distributional test. In: Statistical distribution in scientific work, V. 5, Dordrecht: D. Reidel Publishing Co. 1981.
- SIEGEL, S. Estatística não-paramétrica (para as ciências do comportamento). Editora McGraw-Hill do Brasil, 1977.
- Van Der Auwera, L., Meyer, F., Malet, L.M. The Use of the Weibull 3-Parameters model for estimating mean wind power densities. J. Appl. Meteorol., v. 19, n. 7, p. 819-825, 1980.
- Worley, J.W., Bollinger, J.A., Woeste, F.E., Kline, K.S. Graphic distribution analysis (GDA). American Society of Agricultural Engineers, St. Joseph, v.6, n.3, p.367-371, 1990.