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THE BEAL´S CONJECTURE A GEOMETRIC PROOF

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ABSTRACT

This article has the main goal of present a solution to The Beal Conjecture and Prize, that were widely announced in an article that appeared in the December 1997 issue of Notices of the American Mathematical Society. So far, there is no demonstration for the Conjecture, and the prize money of 1, 000, 000 dollars is being held by the AMS. It is known that Beal's Conjecture is a generalization of Fermat's Last Theorem. By the year of 1993, Fermat s Last Theorem was mathematically demonstrated by Andrew Willes. Nevertheless, the British mathematician used sophisticated concepts and tools, such as modern algebraic geometry, that were not known at Fermat s time. Thus, not yet resigned to that demonstration, we employed basic concepts of algebra and analytic geometry leading to a neater and straighter solution for the Beal´ s conjecture, and yet a simpler resolution. After all, as Da Vinci stated centuries ago, "simplicity is the ultimate sophistication".

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1. INTRODUCTION

The Fermat s last theorem ($a^n = b^n + c^n$) is generalized by Beal's conjecture, that states:

"If $A^X + B^Y = C^Z$, where A, B, C, x, y and z are positive integers and x, y and z are all greater than 2, then A, B and C must have a common prime factor."

In this paper, we will demonstrate the veracity of the conjecture.

2. Argument and proof

Due to methodological reasons, we are going to adopt the following equation for the Beal s conjecture demonstration: $a^x = b^y + c^z$, instead $A^X + B^Y = C^Z$. Thus, A^X stands for c^z; B^Y stands for b^y; and, C^Z stands for a^x.

2.1 Equation notation

If
$$a^x = b^y + c^z \tag{1}$$

it is possible to imply that (1) can assume anyone of the following forms:

$$a^x = \left(b^{\frac{y}{x}}\right)^x + \left(c^{\frac{z}{x}}\right)^x \tag{2}$$
$$\left(a^{\frac{x}{y}}\right)^y = b^y + \left(c^{\frac{z}{y}}\right)^y \tag{3}$$
$$\left(a^{\frac{x}{z}}\right)^z = \left(b^{\frac{y}{z}}\right)^z + c^z \tag{4}$$

And, taking any of the equations (2), (3) or (4), for instance (3), for argumentation and proof s development, we deduce:

$\left(a^{\frac{x}{y}}\right)^{y} > b^{y}$ e $\left(a^{\frac{x}{y}}\right)^{y} > \left(c^{\frac{z}{y}}\right)^{y}$

Or

$a^{\frac{x}{y}} > b$ e $a^{\frac{x}{y}} > c^{\frac{z}{y}}$

Then, the three possibles ways that $a^{\frac{x}{y}}$ is related to the sum of $b + c^{\frac{z}{y}}$ are:

a. $a^{\frac{x}{y}} < b + c^{\frac{z}{y}}$

b. $a^{\frac{x}{y}} = b + c^{\frac{z}{y}}$

c. $a^{\frac{x}{y}} > b + c^{\frac{z}{y}}$

Whence, a), b), c) imply respectively:

$a^{x} < \left(b + c^{\frac{z}{y}}\right)^{y}$, $a^{x} = \left(b + c^{\frac{z}{y}}\right)^{y}$ *and* $a^{x} > \left(b + c^{\frac{z}{y}}\right)^{y}$

Yet, developing the binomial $\left(b + c^{\frac{z}{y}}\right)^{y}$, as result we have:

$$\left(b + c^{\frac{z}{y}}\right)^{y} = b^{y} + \binom{y}{1} \cdot b^{y-1} \cdot c^{\frac{z}{y}} + \cdots + \binom{y}{y-1} \cdot b \cdot \left(c^{\frac{z}{y}}\right)^{y-1} + c^{z}$$

Now, taking the binomial´s development and comparing a), b), c) equations with it, we noticed that only a) and b) are able to solve $a^{x} = b^{y} + c^{z}$ (1). And, it is true, because, c) will always have $a^{x} > b^{y} + c^{z}$ as seen from the binomial $\left(b + c^{\frac{z}{y}}\right)^{y}$ development.

Whence, we realized that there will be (1), if and only if:

$a^{\frac{x}{y}} > b$, $a^{\frac{x}{y}} > c^{\frac{z}{y}}$ *and* $a^{\frac{x}{y}} \leq b + c^{\frac{z}{y}}$

In conclusion, we can set the following order for the tern $a^{\frac{x}{y}}, b, c^{\frac{z}{y}}$.

Thus,

$a^{\frac{x}{y}} > c^{\frac{z}{y}} > b$ or $a^{\frac{x}{y}} > b > c^{\frac{z}{y}}$.

2.2 Argument

Observing the power´s bases from equation (3), we have seen that they must respect $a^{\frac{x}{y}} > b$, $a^{\frac{x}{y}} > c^{\frac{z}{y}}$ and $a^{\frac{x}{y}} \leq b + c^{\frac{z}{y}}$, for the occurrence of (1). Therefore, geometrically, we can say that the tern $a^{\frac{x}{y}}, b, c^{\frac{z}{y}}$ satisfied the relations that define the existence of a triangle, according to the triangle inequality theorem. So, we can establish as premise for our argument, the following geometric logic, transcript on the figure bellow.

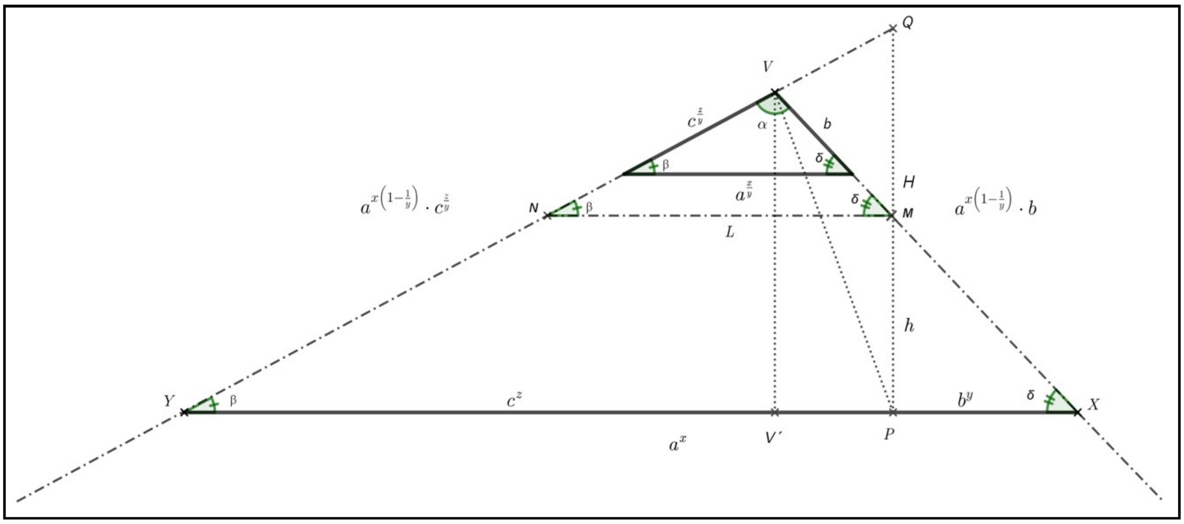

Fig. 1.

And then, for the tern $a^{\frac{x}{y}}, b,\ c^{\frac{z}{y}}$, we make the triangle $\triangle\left(a^{\frac{x}{y}}, b,\ c^{\frac{z}{y}}\right)$, as shown on Fig.1. Next, b and $c^{\frac{z}{y}}$ are extended until it fits the segment a^x parallel to $a^{\frac{x}{y}}$. As a result of it two similar triangles with parallel bases, a^x and $a^{\frac{x}{y}}$ are formed. Thus, by the proportionality statement, the similarity ratio of those two triangles is $a^{x(1-\frac{1}{y})}$. Yet again, by the proportionality statement, it is possible to determine the other two sides, $a^{x(1-\frac{1}{y})}\cdot\ b$ and $a^{x(1-\frac{1}{y})}\cdot c^{\frac{z}{y}}$, of the bigger triangle of base a^x.

On the other hand, if $a^x = b^y + c^z$ (1), then there will be a point *P* which separates b^y from c^z on a^x. And, *P* could or could not be coincident with the orthogonal projection of *V*; that is *V′* on a^x.

2.3 Proof

2.3.1 Hypotheses and resolutions

Suppose exists $a^x = b^y + c^z$ (1), with $(a, b, c) \in \mathbb{N}$ and $(x, y, z) > 2 \in \mathbb{N}$; and, *P* which separates b^y from c^z on a^x. So, it could or could not to be coincident with the orthogonal projection of $V \rightarrow V'$.

Situation – I

$$b^y > a^{x(1-\frac{1}{y})}\cdot\ b\ \cdot\cos\delta \quad (5)$$

and $c^z < a^{x(1-\frac{1}{y})}\cdot c^{\frac{z}{y}}\cdot\ \cos\beta$ (6)

If and only if, *P* is on the left of *V′*.

Situation – II

$$b^y = a^{x(1-\frac{1}{y})}\cdot\ b\cdot\ \cos\delta \quad (7)$$

and $c^z = a^{x(1-\frac{1}{y})}\cdot\ c^{\frac{z}{y}}\ \cdot\cos\beta$ (8)

If and only if, *P* is coincident with *V′*.

Situation – III

$$b^y < a^{x(1-\frac{1}{y})}\ \cdot b\ \cdot\cos\delta \quad (9)$$

and $c^z > a^{x(1-\frac{1}{y})}\cdot c^{\frac{z}{y}}\cdot\ \cos\beta$ (10)

If and only if, *P* is on the right of *V`*.

Regardless of the presented situations, we will have:

$$\cos\delta = \frac{\left(a^{\frac{x}{y}}\right)^2 - \left[\left(c^{\frac{z}{y}}\right)^2 - b^2\right]}{2a^{\frac{x}{y}}\cdot b} \qquad \text{and} \qquad \cos\beta = \frac{\left(a^{\frac{x}{y}}\right)^2 + \left[\left(c^{\frac{z}{y}}\right)^2 - b^2\right]}{2a^{\frac{x}{y}}\cdot\ c^{\frac{z}{y}}}$$

And, those are the demonstrations for each situation.

2.3.2 Proof: situation I

If $\cos\delta$ and $\cos\beta$ are replaced in (5) and (6), it results:

$$b^y > a^{x(1-\frac{1}{y})}\ \cdot b\cdot\frac{\left(a^{\frac{x}{y}}\right)^2 - \left[\left(c^{\frac{z}{y}}\right)^2 - b^2\right]}{2a^{\frac{x}{y}}\cdot b} \qquad \text{and} \qquad c^z < a^{x(1-\frac{1}{y})}\cdot\ c^{\frac{z}{y}}\ \cdot\frac{\left(a^{\frac{x}{y}}\right)^2 + \left[\left(c^{\frac{z}{y}}\right)^2 - b^2\right]}{2a^{\frac{x}{y}}\cdot\ c^{\frac{z}{y}}}$$

Or

$$-b^y < -a^{\left(x-\frac{x}{y}\right)}\ \cdot\frac{\left(a^{\frac{x}{y}}\right)^2 - \left[\left(c^{\frac{z}{y}}\right)^2 - b^2\right]}{2a^{\frac{x}{y}}} \qquad \text{and} \qquad c^z < a^{\left(x-\frac{x}{y}\right)}\ \cdot\frac{\left(a^{\frac{x}{y}}\right)^2 + \left[\left(c^{\frac{z}{y}}\right)^2 - b^2\right]}{2a^{\frac{x}{y}}}$$

Whence,

$$c^z - b^y < a^{(x-\frac{x}{y})}\ \frac{-\left(a^{\frac{x}{y}}\right)^2 + \left[\left(c^{\frac{z}{y}}\right)^2 - b^2\right] + \left[\left(c^{\frac{z}{y}}\right)^2 - b^2\right] + \left(a^{\frac{x}{y}}\right)^2}{2a^{\frac{x}{y}}}$$

or

$$c^z - b^y < a^{(x-\frac{x}{y})}\ \frac{2\cdot\left[\left(c^{\frac{z}{y}}\right)^2 - b^2\right]}{2a^{\frac{x}{y}}}$$

Thus

$$c^z - b^y < a^{(x-\frac{2x}{y})} \left[\left(c^{\frac{z}{y}}\right)^2 - b^2\right]$$

and as result

$$a^{x\left(1-\frac{2}{y}\right)} > \frac{c^z-b^y}{\left(c^{\frac{z}{y}}\right)^2-b^2}$$

or

$$a^{x\left(1-\frac{2}{y}\right)} > \frac{\left(c^{\frac{z}{y}}\right)^y-b^y}{\left(c^{\frac{z}{y}}\right)^2-b^2} \qquad (11)$$

Next, effecting the division on the second member of the inequality (11):

$$a^{x\left(1-\frac{2}{y}\right)} > \left(c^{\frac{z}{y}}\right)^{y-2} + \left(c^{\frac{z}{y}}\right)^{y-4} b^2 + \frac{\left(c^{\frac{z}{y}}\right)^{y-4} b^4-b^y}{\left(c^{\frac{z}{y}}\right)^2-b^2}$$

Then, when both members of the inequality above is multiplied by $\left(a^{\frac{x}{y}}\right)^2$, we will have:

$$a^x > \left(a^{\frac{x}{y}}\right)^2\left(c^{\frac{z}{y}}\right)^{y-2} + \left(a^{\frac{x}{y}}\right)^2\left(c^{\frac{z}{y}}\right)^{y-4} \cdot b^2 + \left(a^{\frac{x}{y}}\right)^2 \cdot \frac{\left(c^{\frac{z}{y}}\right)^{y-4} b^4 - b^y}{\left(c^{\frac{z}{y}}\right)^2 - b^2}$$

But,

$$\left(a^{\frac{x}{y}}\right)^2\left(c^{\frac{z}{y}}\right)^{y-2} > \left(c^{\frac{z}{y}}\right)^y \qquad \text{and} \qquad \left(a^{\frac{x}{y}}\right)^2\left(c^{\frac{z}{y}}\right)^{y-4} \cdot b^2 > b^y$$

And, since $\left(a^{\frac{x}{y}}\right) > \left(c^{\frac{z}{y}}\right) > b$ was established as premise, hence:

$\left(a^{\frac{x}{y}}\right)^y > b^y + \left(c^{\frac{z}{y}}\right)^y$ that is, $a^x > b^y + c^z$

Whenever the point *P* (yet again, that separates b^y from c^z) is on the left side of *V´*. Thus, it is proved the nonexistence of $a^x = b^y + c^z$ (1) for the situation – I.

2.3.3 Proof: situation II

From (7) and (8):

$$b^y = a^{x(1-\frac{1}{y})} \cdot b \cdot \cos\delta \qquad \text{and} \qquad c^z = a^{x(1-\frac{1}{y})} \cdot c^{\frac{z}{y}} \cdot \cos\beta\ ,$$

and, by subtracting $c^z - b^y$, it results:

$$c^z - b^y = a^{x(1-\frac{1}{y})} \cdot \left(c^{\frac{z}{y}} \cdot \cos\beta - b \cdot \cos\delta\right),$$

where

$$\cos\beta = \frac{\left(a^{\frac{x}{y}}\right)^2 + \left[\left(c^{\frac{z}{y}}\right)^2 - b^2\right]}{2a^{\frac{x}{y}} \cdot c^{\frac{z}{y}}} \quad \text{and} \quad \cos\delta = \frac{\left(a^{\frac{x}{y}}\right)^2 - \left[\left(c^{\frac{z}{y}}\right)^2 - b^2\right]}{2a^{\frac{x}{y}} \cdot b}\ ,$$

and, when these cosines are inserted in the equation, we have:

$$a^{x(1-\frac{2}{y})} = \frac{c^z-b^y}{\left(c^{\frac{z}{y}}\right)^2-b^2} \quad \text{or} \quad a^{x(1-\frac{2}{y})} = \frac{\left(c^{\frac{z}{y}}\right)^y-b^y}{\left(c^{\frac{z}{y}}\right)^2-b^2}.$$

This result is like the one from situation – I, inequality (11). Obviously, switching the signal from “>” to the “=”, since this result is an equation and not an inequality.

So, it implies:

$$a^x = \left(a^{\frac{x}{y}}\right)^2\left(c^{\frac{z}{y}}\right)^{y-2} + \left(a^{\frac{x}{y}}\right)^2\left(c^{\frac{z}{y}}\right)^{y-4} \cdot b^2 + \left(a^{\frac{x}{y}}\right)^2 \cdot \frac{\left(c^{\frac{z}{y}}\right)^{y-4} b^4-b^2}{\left(c^{\frac{z}{y}}\right)^2-b^2}.$$

But, since

$$\left(a^{\frac{x}{y}}\right)^2\left(c^{\frac{z}{y}}\right)^{y-2} > \left(c^{\frac{z}{y}}\right)^{y} \quad \text{and} \quad \left(a^{\frac{x}{y}}\right)^2\left(c^{\frac{z}{y}}\right)^{y-4} \cdot b^2 > b^y ,$$

then, and again, it is to be concluded that $a^x > \left(c^{\frac{z}{y}}\right)^y + b^y$. What is absurd, once that by hypothesis $a^x = b^y + c^z$. Therefore, remains $a^x > b^y + c^z$, every time that *P* coincides with *V'*.

And yet, it is proved the nonexistence $a^x = b^y + c^z$ (1) for the situation – II.

2.3.4 Proof: situation III

Once again, returning to Fig. 1, consider the similar triangles $\triangle (V, X, Y) \approx \triangle (V, M, N)$, from where:

$$\frac{\overline{VM}}{\overline{VX}} = \frac{\overline{MN}}{\overline{XY}} \quad \text{or} \quad \frac{\overline{VX}-\overline{XM}}{\overline{VX}} = \frac{\overline{MN}}{\overline{XY}} .$$

But, as

$$\cos\delta = \frac{\overline{XP}}{\overline{XM}} \quad \text{or} \quad \overline{XM} = \frac{\overline{XP}}{\cos\delta} .$$

So,

$$\frac{\overline{VX}-\frac{\overline{XP}}{\text{co}}}{\overline{VX}} = \frac{\overline{MN}}{\overline{XY}} \tag{12}$$

However,

$$\overline{VX} = a^{x(1-\frac{1}{y})} \cdot b,\ \overline{XP} = b^y,\ \overline{MN} = L \quad \text{and} \quad \overline{XY} = a^x .$$

Hence, (12) can be written as:

$$\frac{a^{x(1-\frac{1}{y})} \cdot b - \frac{b^y}{\cos\delta}}{a^{x(1-\frac{1}{y})} \cdot b} = \frac{L}{a^x} \quad \text{or} \quad \frac{a^{x(1-\frac{1}{y})} b \cdot \cos\delta - b^y}{a^{x(1-\frac{1}{y})} \cdot b \cdot \cos\delta} = \frac{L}{a^x} .$$

So,

$$L = \frac{a^{(2x-\frac{x}{y})} \cdot \cos\delta - {}^{x} \cdot b^{y-1}}{a^{(x-\frac{x}{y})} \cdot \cos\delta} ,$$

then

$$L = \frac{a^{(2x-\frac{x}{y}-x+\frac{x}{y})} \cdot \cos\delta - {}^{(x-x+\frac{x}{y})} \cdot b^{y-1}}{\cos\delta} ,$$

whence

$$L = \frac{a^{x} \cdot \cos\delta - a^{\frac{x}{y}} \cdot b^{y-1}}{\cos\delta} \tag{13}$$

So, returning to Fig. 1; and, now considering the similar triangles $\triangle (Q, P, Y) \approx \triangle (Q, M, N)$ we have:

$$\frac{\overline{QP}}{\overline{PY}} = \tan\beta \quad \text{and} \quad \frac{\overline{MP}}{\overline{PX}} = \tan\delta,$$

And, by the triangle similarity principles, we have:

$$\frac{\overline{QM}}{\overline{QP}} = \frac{\overline{MN}}{\overline{PY}}$$

But, since

$$\overline{QM} = \overline{QP} - \overline{MP} \quad \text{and} \quad \overline{MN} = L ,$$

then

$$\frac{\overline{QP}-\overline{MP}}{\overline{QP}} = \frac{L}{\overline{PY}} .$$

Also, it is to be noted that

$$\overline{QP} = H ,\quad \overline{MP} = h \quad \text{and} \quad \overline{PY} = c^z ,$$

Then,

$$\frac{H-h}{H} = \frac{L}{c^z}$$

And, if

$\frac{H}{c^z} = \tan\beta$ and $\frac{h}{b^y} = \tan\delta$,

Then

$\frac{c^z \tan\beta \quad {}^y \tan\delta}{c^z \tan\beta} = \frac{L}{c^z}$,

Whence

$$L = \frac{c^z \tan\beta - b^y \tan\delta}{\tan\beta} \tag{14}$$

Therefore, continuing into the proof development, we compare (13) and (14), and it results in:

$$\frac{a^x \cdot \cos\delta - {}^{\frac{x}{y}} \cdot b^{y-1}}{\cos\delta} = \frac{c^z \cdot \tan\beta - b^y \cdot \tan\delta}{\tan}.$$

And, when we substitute $\tan\beta$ and $\tan\delta$, we come to:

$$\frac{a^x \cdot \cos\delta - a^{\frac{x}{y}} \cdot b^{y-1}}{\cos\delta} = \frac{\frac{c^z \cdot \sin}{\cos\beta} - \frac{b^y \cdot \sin\delta}{\cos\delta}}{\frac{\sin\beta}{\cos\beta}} ,$$

or

$$\frac{a^x \cdot \cos\delta - a^{\frac{x}{y}} \cdot b^{y-1}}{\cos\delta} = \frac{\frac{c^z \cdot \sin\beta \cdot \cos\delta - b^y \cdot \sin \quad \cdot \cos\beta}{\cos\beta \cdot \cos\delta}}{\frac{\sin\beta}{\cos\beta}} .$$

So,

$$\frac{a^x \cdot \cos\delta - a^{\frac{x}{y}} \cdot b^{y-1}}{\cos\delta} = \frac{c^z \cdot \sin\beta \cdot \cos\delta - b^y \cdot \sin\delta \cdot \cos\beta}{\cos\beta \cos\delta} \cdot \frac{\cos\beta}{\sin\beta} ,$$

Whence

$$\frac{a^x \cdot \cos\delta - a^{\frac{x}{y}} \cdot b^{y-1}}{\cos\delta} = \frac{c^z \cdot \sin\beta \cdot \cos\delta - b^y \cdot \sin\delta \cdot \cos\beta}{\sin\beta \cdot \cos\delta} ,$$

then

$$a^x \cdot \cos\delta - a^{\frac{x}{y}} \cdot b^{y-1} = \frac{c^z \cdot \sin\beta \cdot \cos\delta - b^y \cdot \sin\delta \cdot \cos\beta}{\sin\beta} ,$$

Hence

$$a^x \cdot \sin\beta \cdot \cos\delta - a^{\frac{x}{y}} \cdot b^{y-1} \cdot \sin\beta = c^z \cdot \sin\beta \cdot \cos\delta - b^y \cdot \sin\delta \cdot \cos\beta.$$

Since, from the hypothesis $a^x = b^y + c^z$ (1), then,

$$(b^y + c^z) \cdot \sin\beta \cdot \cos\delta - a^{\frac{x}{y}} \cdot b^{y-1} \cdot \sin\beta = c^z \cdot \sin\beta \cdot \cos\delta - b^y \cdot \sin\delta \cdot \cos\beta ,$$

or

$$b^y \cdot \sin\beta \cdot \cos\delta - a^{\frac{x}{y}} \cdot b^{y-1} \cdot \sin\beta = -b^y \cdot \sin\delta \cdot \cos\beta .$$

So,

$$b^y \cdot (\sin\beta \cdot \cos\delta + \sin\delta \cdot \cos\beta) = a^{\frac{x}{y}} \cdot b^{y-1} \cdot \sin\beta ,$$

thus,

$$b^y \cdot \sin(\beta + \delta) = a^{\frac{x}{y}} \cdot b^{y-1} \cdot \sin\beta,$$

or yet

$$b \cdot \sin(\beta + \delta) = a^{\frac{x}{y}} \cdot \sin\beta.$$

Therefore,

$$\frac{a^{\frac{x}{y}}}{\sin(\beta+\delta)} = \frac{b}{\sin\beta} \tag{15}.$$

However, from $\triangle\left(a^{\frac{x}{y}}, b, c^{\frac{z}{y}}\right)$, Fig – 1, and by the law of sines, the outcome is:

$$\frac{a^{\frac{x}{y}}}{\sin\alpha} = \frac{b}{\sin\beta} = \frac{c^{\frac{z}{y}}}{\sin\delta}$$

where

$$\frac{a^{\frac{x}{y}}}{\sin\alpha} = \frac{b}{\sin\beta} \quad (16)$$

Comparing (15) with (16), it results:

$$\frac{a^{\frac{x}{y}}}{\sin(\beta+\delta)} = \frac{a^{\frac{x}{y}}}{\sin\alpha} \ ,$$

which implies that $\sin\alpha = \sin(\beta+\delta)$, or $\alpha = \beta + \delta$.

And, since $\alpha + \beta + \delta = 180°$, therefore, $2\beta + 2\delta = 180°$; or yet, $\beta + \delta = 90°$.

Whence, $\alpha = 90°$.

As consequence, it is mandatory that the triangle $\triangle\left(a^{\frac{x}{y}}, b,\ c^{\frac{z}{y}}\right)$, Fig – 1, is a rectangle triangle, which implies:

$$\left(a^{\frac{x}{y}}\right)^2 = b^2 + \left(c^{\frac{z}{y}}\right)^2 \quad (17)$$

Thus, the equation (17) is equivalent to the equation (1). That is, the exponents of both equations, (1) and (17), are equivalents.

Yet, $\frac{2x}{y} = x$, $2 = y$ and $\frac{2z}{y} = z$.

Therefore, taking $y = 2$ into (17), we have:

$$a^x = b^2 + c^z \quad (18)$$

In the same way, when we consider the equations (2) and (4), it results, respectively:

$a^2 = b^y + c^z$ and $a^x = b^y + c^2$.

So, always that (x, y, z) are solution for $a^x = b^y + c^z$ (1), at least one of them will be equals 2. Obviously, we are not considering the trivial solution in which one of the elements is equals 1.

Finally, remains proved that for the situation – III, where $(a, b, c, x, y, z) \in \mathbb{N}$, $a^x = b^y + c^z$ (1), will never be possible if $y > 2$, como afirma Beal.

CONCLUSION

If it is considered the equation $a^x = b^y + c^z$ (1), which there will be three and only three possibilities for the position of the point *P*, which separates b^y from c^z on the a^x as shown at Fig – 1; then, it implies in the three situations here investigated. The situations I, II and III were able to prove the truthiness of Beal´s Conjecture, as seen.

For the situation – I, it is proved the nonexistence of (1), because, $a^x > b^y + c^z$ when $y > 2$.

About the situation – II, under the same conditions, it is repeated the nonexistence of (1). That is, $a^x > b^y + c^z$.
At last, analysing the situation – III, considered (3), we were able to prove the outcome in which $a^x = b^2 + c^z$ (18).
Therefore, and as seen, there is nothing else to prove. But, to announce the truthiness fo Beal´s Conjecture. Q.E.D.

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